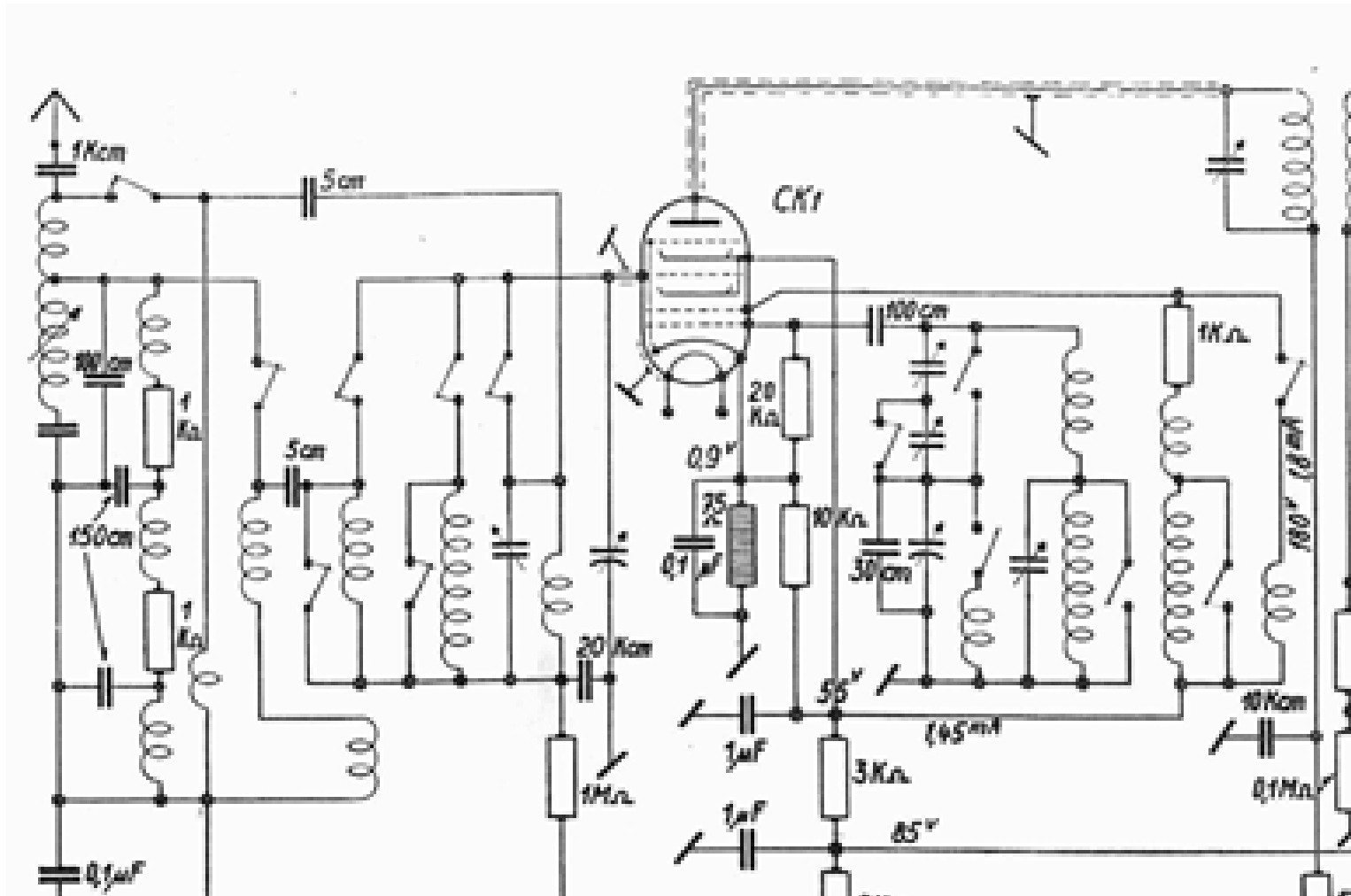
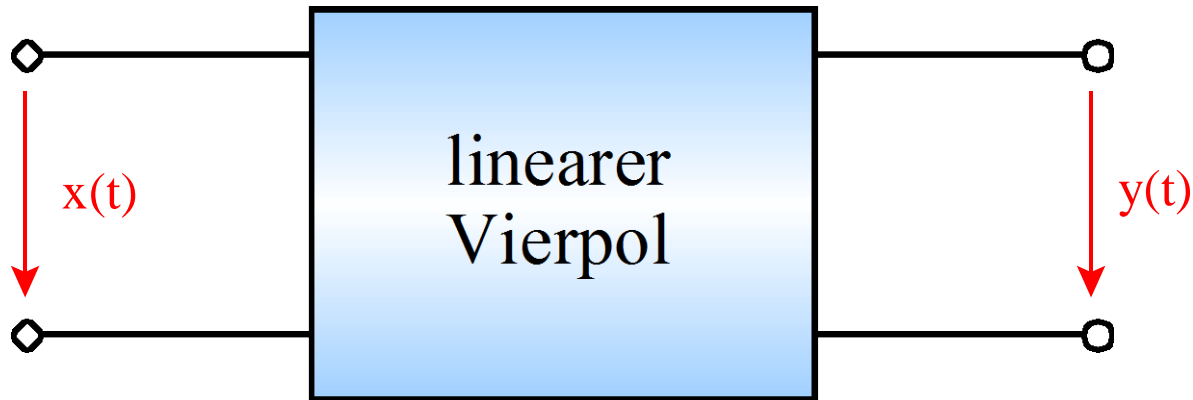


# 7. Ausgewählte Wechselstromanordnungen



## 7. Ausgewählte Wechselstromanordnungen

### 7.1 Schaltungen mit frequenzselektiven Eigenschaften



Zeitbereich:

$$y(t) = f\left(x(t), \frac{dx}{dt}, \int x(t)dt\right)$$

Bildbereich (komplexe Ebene):

$$\underline{\hat{Y}}(j\omega) = \underline{H}(j\omega) \underline{\hat{X}}(j\omega)$$

Übertragungsfunktion:

$$\underline{H}(j\omega) = \frac{\underline{\hat{Y}}(j\omega)}{\underline{\hat{X}}(j\omega)}$$

$$\underline{\hat{Y}}(j\omega) = \underline{H}(j\omega) \underline{\hat{X}}(j\omega)$$

$$\hat{Y}(\omega) e^{j\varphi_y(\omega)} = H(\omega) e^{j\varphi_H(\omega)} \hat{X}(\omega) e^{j\varphi_x(\omega)}$$

Amplitudenübertragung:

$$\hat{Y}(\omega) = H(\omega) \hat{X}(\omega)$$

**Amplitudenfrequenzgang des Systems**

Phasenübertragung:

$$\varphi_y(\omega) = \varphi_H(\omega) + \varphi_x(\omega)$$

**Phasenfrequenzgang des Systems**

Amplitudenübertragung:

$$\hat{Y}(\omega) = H(\omega) \hat{X}(\omega)$$

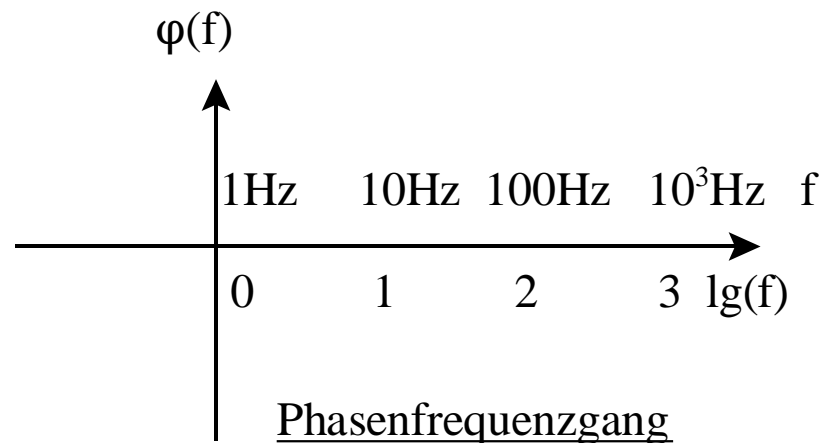
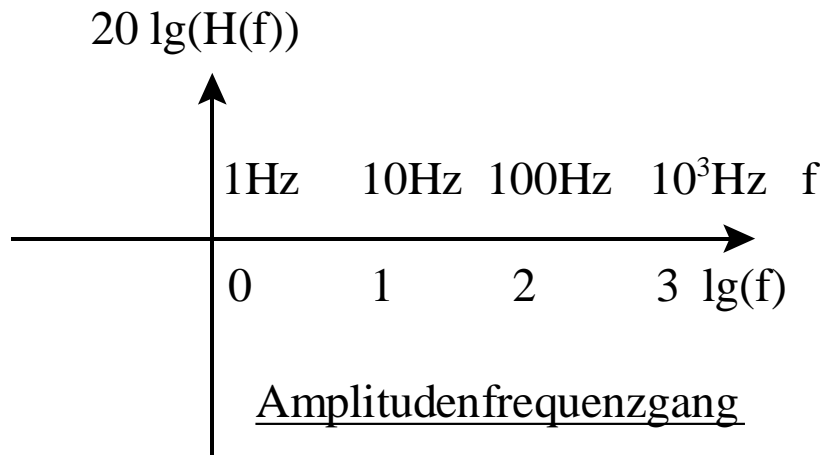
Phasenübertragung:

$$\varphi_y(\omega) = \varphi_H(\omega) + \varphi_x(\omega)$$

$$\lg(\hat{Y}(\omega)) = \lg(H(\omega)) + \lg(\hat{X}(\omega))$$

$$20\lg(\hat{Y}(\omega)) = 20\lg(H(\omega)) + 20\lg(\hat{X}(\omega)) \quad \text{in dB}$$

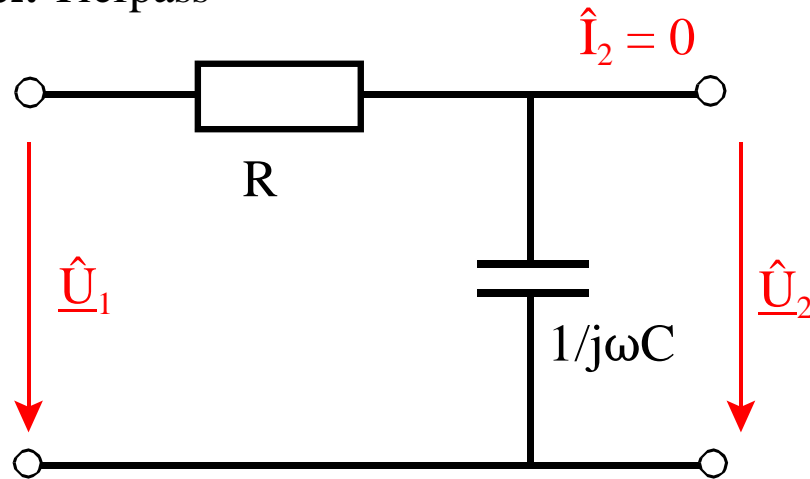
- übliche grafische Darstellung:



## 7.7.1 RC- und RL- Schaltungen

### a) RC-Tiefpass

**Beispiel:** Tiefpass



RC - Glied als Tiefpaß

$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{1}{R + \frac{1}{j\omega C}}$$

$$\underline{H}(j\omega) = \frac{\underline{\hat{U}}_2(j\omega)}{\underline{\hat{U}}_1(j\omega)} = \frac{1}{1 + j\omega CR}$$

Amplitudenfrequenzgang:

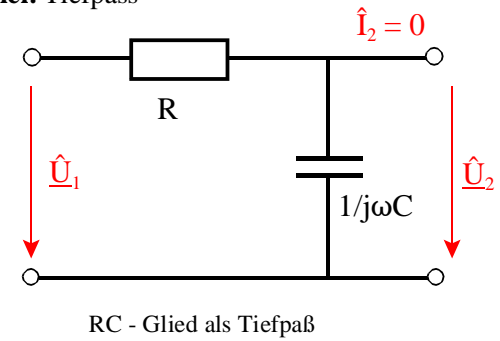
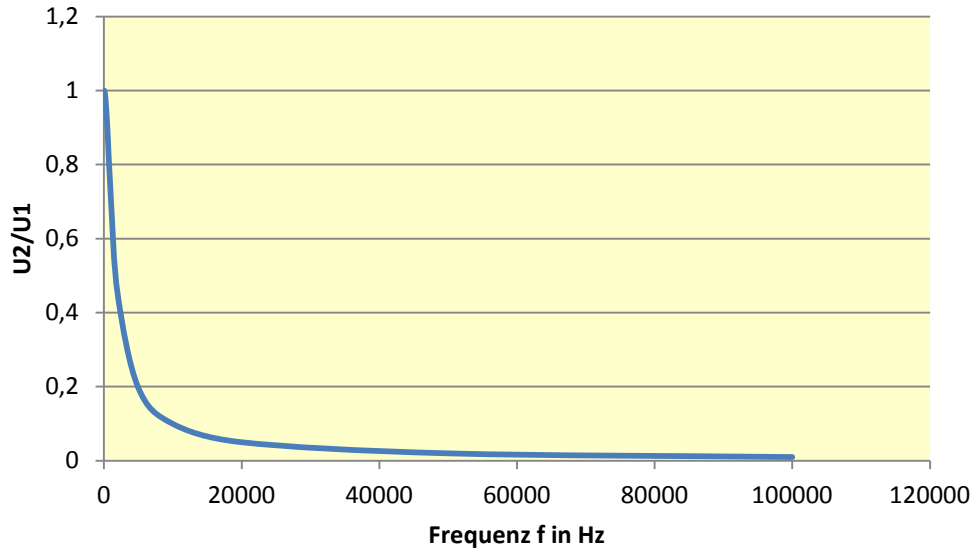
$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

Phasenfrequenzgang:

$$\varphi_H(\omega) = \varphi_{u_2}(\omega) - \varphi_{u_1}(\omega) = -\arctan \omega CR$$

### Beispiel: Tiefpaß

## Lineare Darstellung des Amplitudengangs



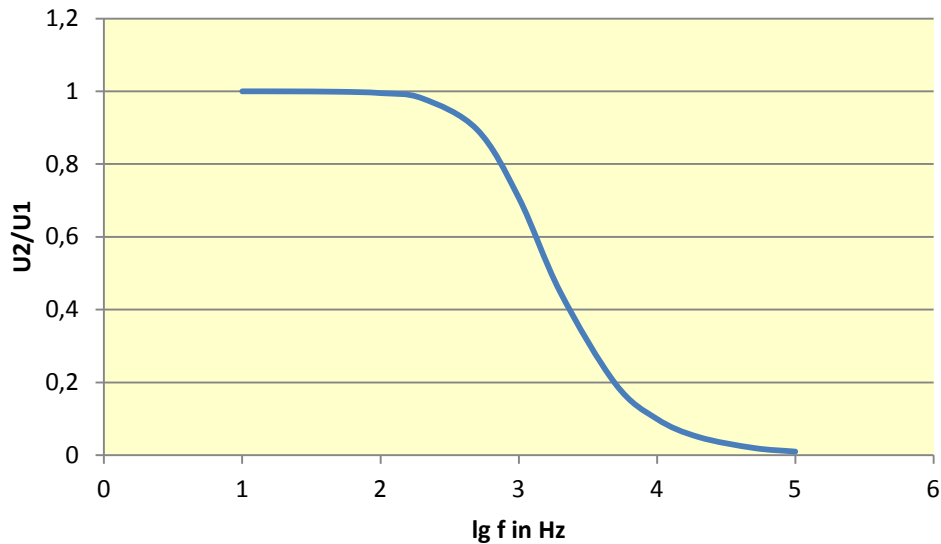
RC - Glied als Tiefpaß

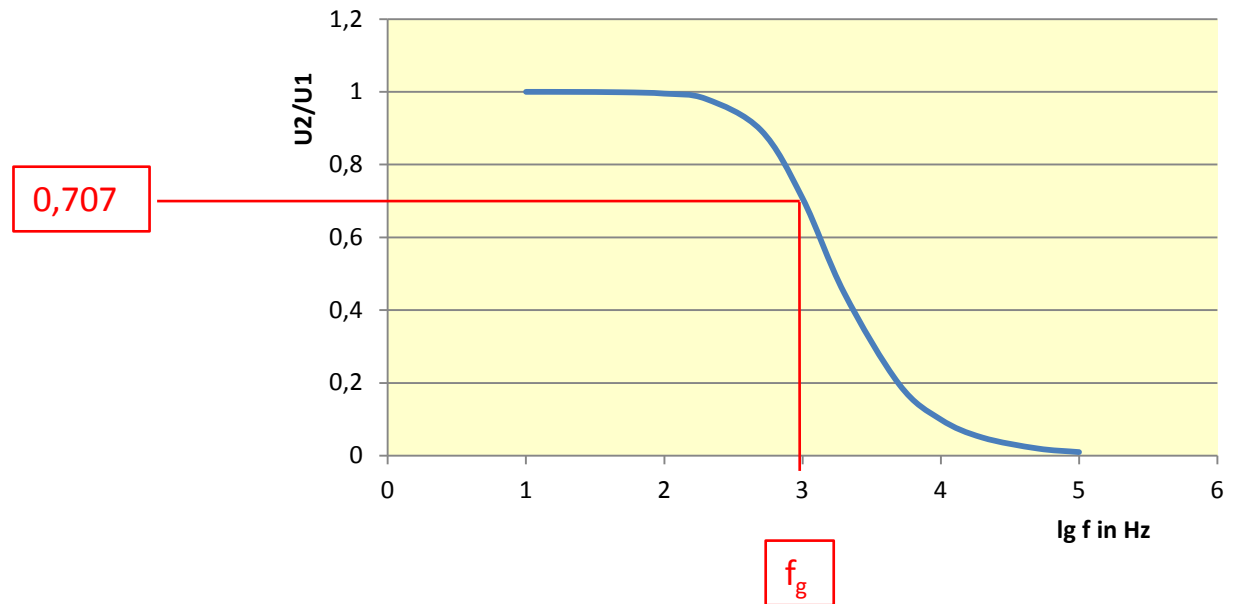
Werte:  
 $R=159 \Omega$   
 $C=1 \mu\text{F}$

### Amplitudenfrequenzgang:

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

## Logarithmische Darstellung des Amplitudengangs





Grenzfrequenz:

$\omega_g$  oder  $f_g$  mit  $\omega_g = 2\pi f_g$

Definition:  $f_g$  ist jene Frequenz, bei der  $|H(\omega)|$  auf den  $\frac{1}{\sqrt{2}} = 2^{-\frac{1}{2}}$ -fachen Wert

des Maximalwertes abgesunken ist oder der Phasenwinkel  $\varphi(\omega) = 45^\circ$  beträgt oder der Realteil von  $\underline{H}(j\omega) =$  Imaginärteil von  $\underline{H}(j\omega)$  ist.

$$\underline{H}(j\omega) = \frac{\hat{U}_2(j\omega)}{\hat{U}_1(j\omega)} = \frac{1}{1 + j\omega CR} \quad \Rightarrow \quad \omega_g RC = 1 \text{ oder } \omega_g = \frac{1}{RC} = 2 \cdot \pi \cdot f_g = \frac{1}{\tau}$$

Grafische Darstellung des Amplitudengangs  
mit Hilfsgeraden:

$$20 \lg(\hat{Y}(\omega)) = 20 \lg(H(\omega)) + 20 \lg(\hat{X}(\omega)) \quad \text{in dB}$$

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + (\omega CR)^2}}$$

$$20 \cdot \lg[H(\omega)] = -10 \cdot \lg[1 + (\omega RC)^2] = -10 \cdot \lg \left[ 1 + \left( \frac{\omega}{\omega_g} \right)^2 \right]$$

$$\omega_g = \frac{1}{RC}$$

oder

$$20 \cdot \lg[H(f)] = -10 \cdot \lg \left[ 1 + \left( \frac{f}{f_g} \right)^2 \right]$$

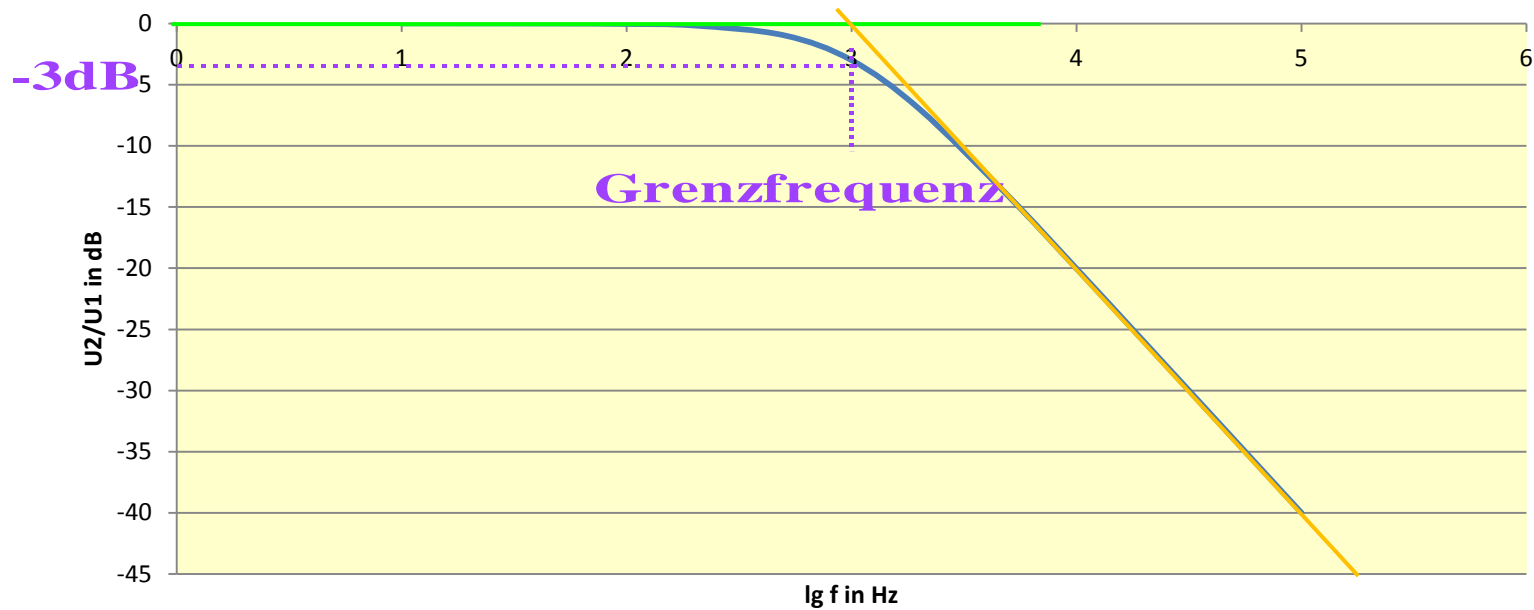
für  $\frac{f}{f_g} \ll 1$  gilt:  $20 \cdot \lg[H(f)] = -10 \cdot \lg \left[ 1 + \left( \frac{f}{f_g} \right)^2 \right] \approx 0$

für  $\frac{f}{f_g} \gg 1$  gilt:  $20 \cdot \lg[H(f)] \approx -20 \cdot \lg \left[ \left( \frac{f}{f_g} \right) \right] = -20 \cdot \lg f + 20 \cdot \lg f_g$



$$\frac{f}{f_g} \ll 1 \quad 20 \cdot \lg[H(f)] \approx 0$$

$$\frac{f}{f_g} \gg 1 \quad 20 \cdot \lg[H(f)] \approx -20 \cdot \lg f + 20 \cdot \lg f_g$$

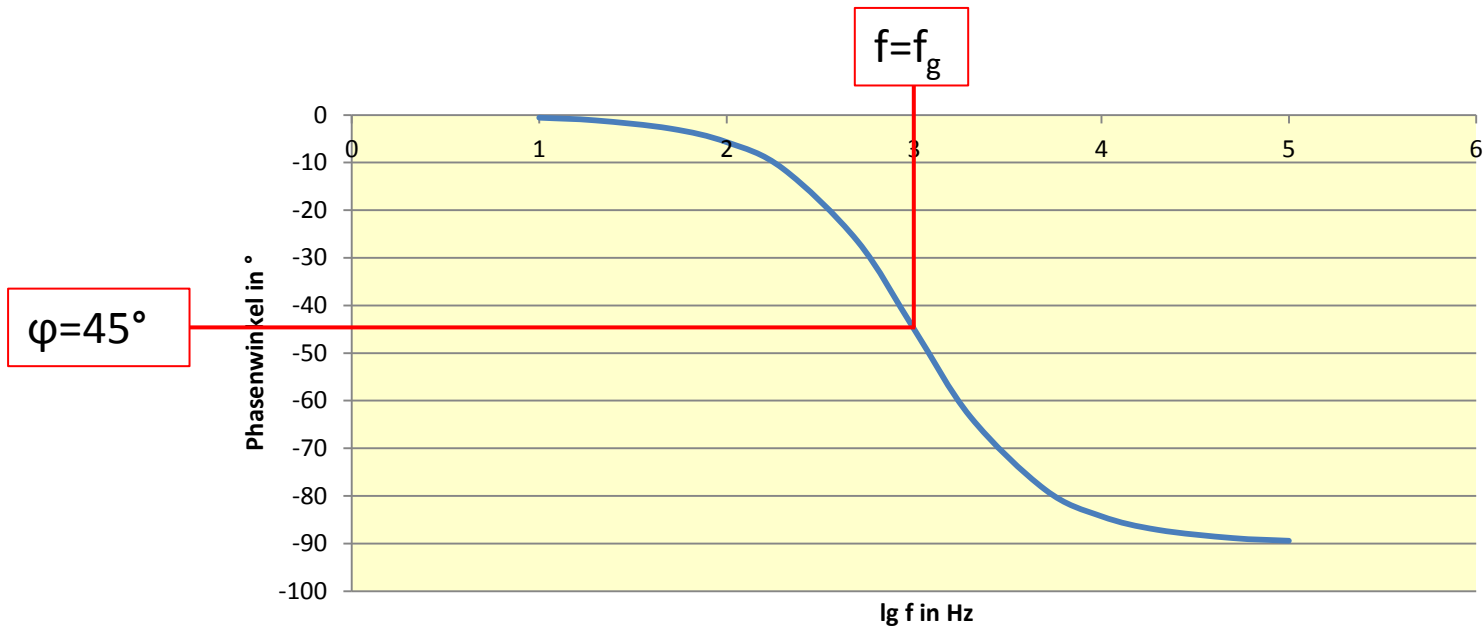


Grafische Darstellung des Phasengangs:

$$\varphi_H(\omega) = -\arctan \omega CR$$

$$\varphi_H(\omega) = -\arctan\left(\frac{\omega}{\omega_g}\right) \quad \text{oder} \quad \varphi_H(f) = -\arctan\left(\frac{f}{f_g}\right)$$

$$\omega_g = \frac{1}{RC}$$

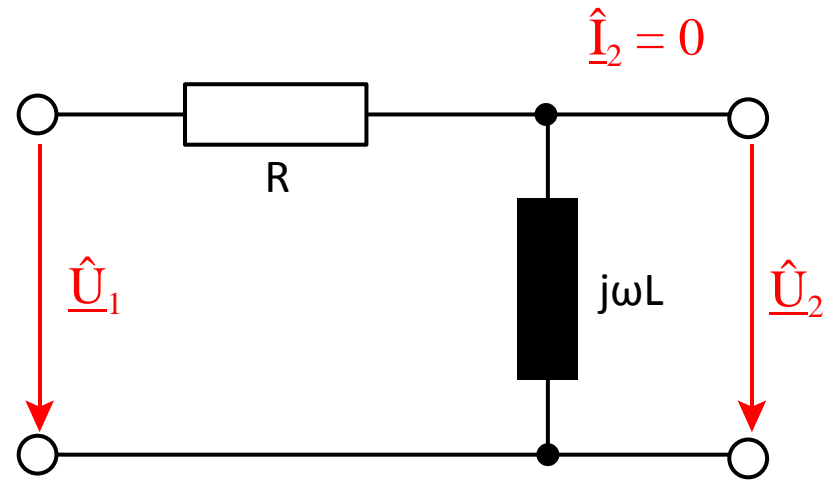


b) R-L-Hochpass:

$$\begin{aligned} \underline{H}(j\omega) &= \frac{\hat{U}_2(j\omega)}{\hat{U}_1(j\omega)} = \frac{j\omega L}{R + j\omega L} \\ &= \frac{1}{1 + \frac{R}{j\omega L}} = \frac{1}{1 - j\frac{R}{\omega L}} \end{aligned}$$

Grenzfrequenz:

$$\frac{R}{\omega_g L} = 1 \quad \omega_g = \frac{R}{L} = \frac{1}{\tau}$$



Amplitudengang:

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}}$$

Phasengang:

$$\varphi_H(\omega) = \varphi_{U_2}(\omega) - \varphi_{U_1}(\omega)$$

$$\varphi_H(\omega) = 0 + \arctan\left(\frac{R}{\omega L}\right)$$

Grafische Darstellung des Amplitudengangs  
mit Hilfsgeraden:

$$20 \lg(\hat{Y}(\omega)) = 20 \lg(H(\omega)) + 20 \lg(\hat{X}(\omega)) \quad \text{in dB}$$

$$H(\omega) = \frac{\hat{U}_2(\omega)}{\hat{U}_1(\omega)} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}}$$

$$20 \cdot \lg[H(\omega)] = -10 \cdot \lg\left[1 + \left(\frac{R}{\omega L}\right)^2\right] = -10 \cdot \lg\left[1 + \left(\frac{\omega_g}{\omega}\right)^2\right] \quad \omega_g = \frac{R}{L}$$

oder

$$20 \cdot \lg[H(f)] = -10 \cdot \lg\left[1 + \left(\frac{f_g}{f}\right)^2\right]$$

für  $\frac{f_g}{f} \ll 1$  gilt:  $20 \cdot \lg[H(f)] = -10 \cdot \lg\left[1 + \left(\frac{f_g}{f}\right)^2\right] \approx 0$

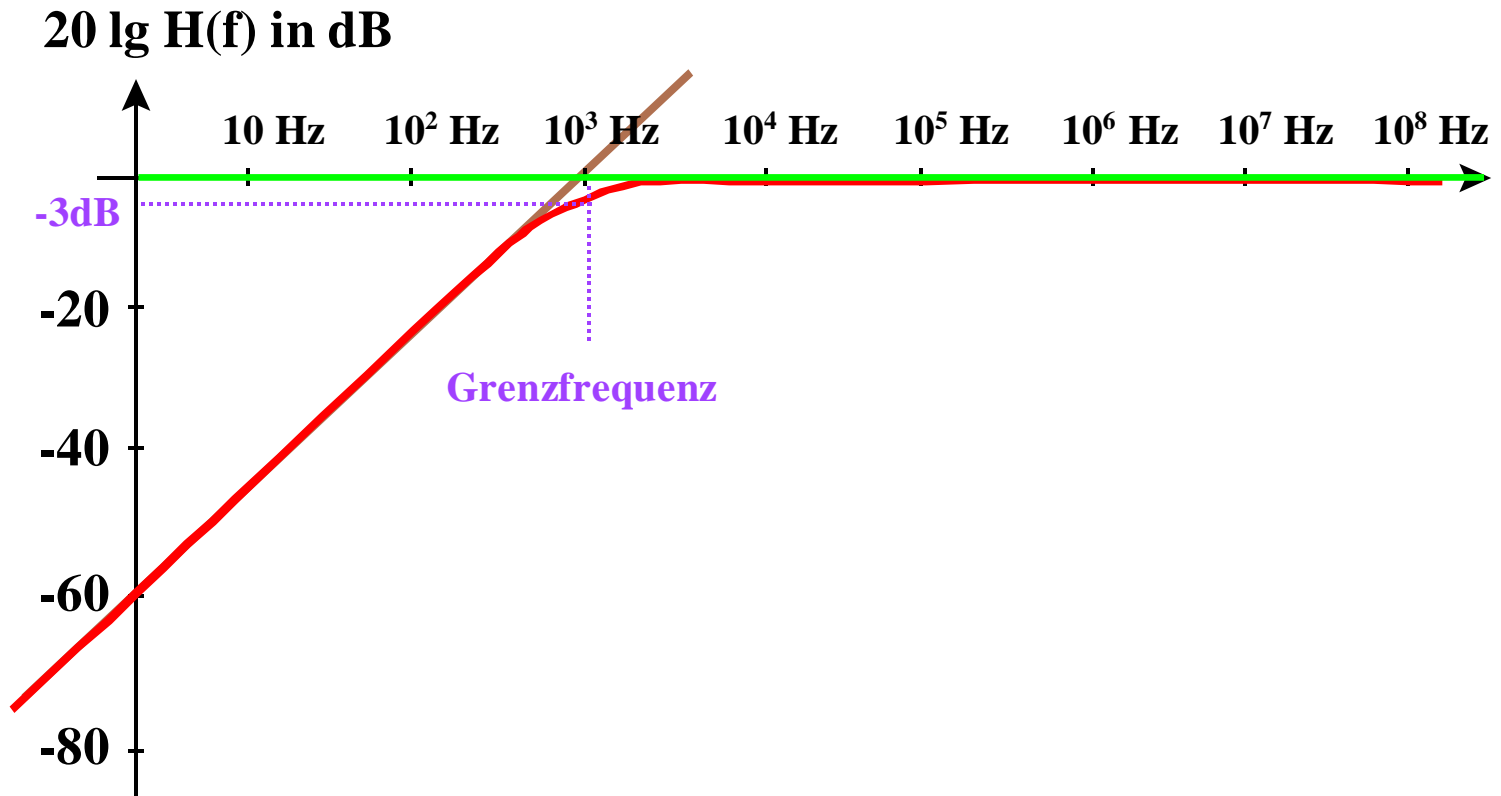
für  $\frac{f_g}{f} \gg 1$  gilt:  $20 \cdot \lg[H(f)] \approx -20 \cdot \lg\left[\left(\frac{f_g}{f}\right)\right] = +20 \cdot \lg f - 20 \cdot \lg f_g$

$$\frac{f_g}{f} \ll 1: \quad 20 \cdot \lg[H(f)] \approx 0$$

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$$\frac{f_g}{f} \gg 1: \quad 20 \cdot \lg[H(f)] \approx +20 \cdot \lg f - 20 \cdot \lg f_g$$

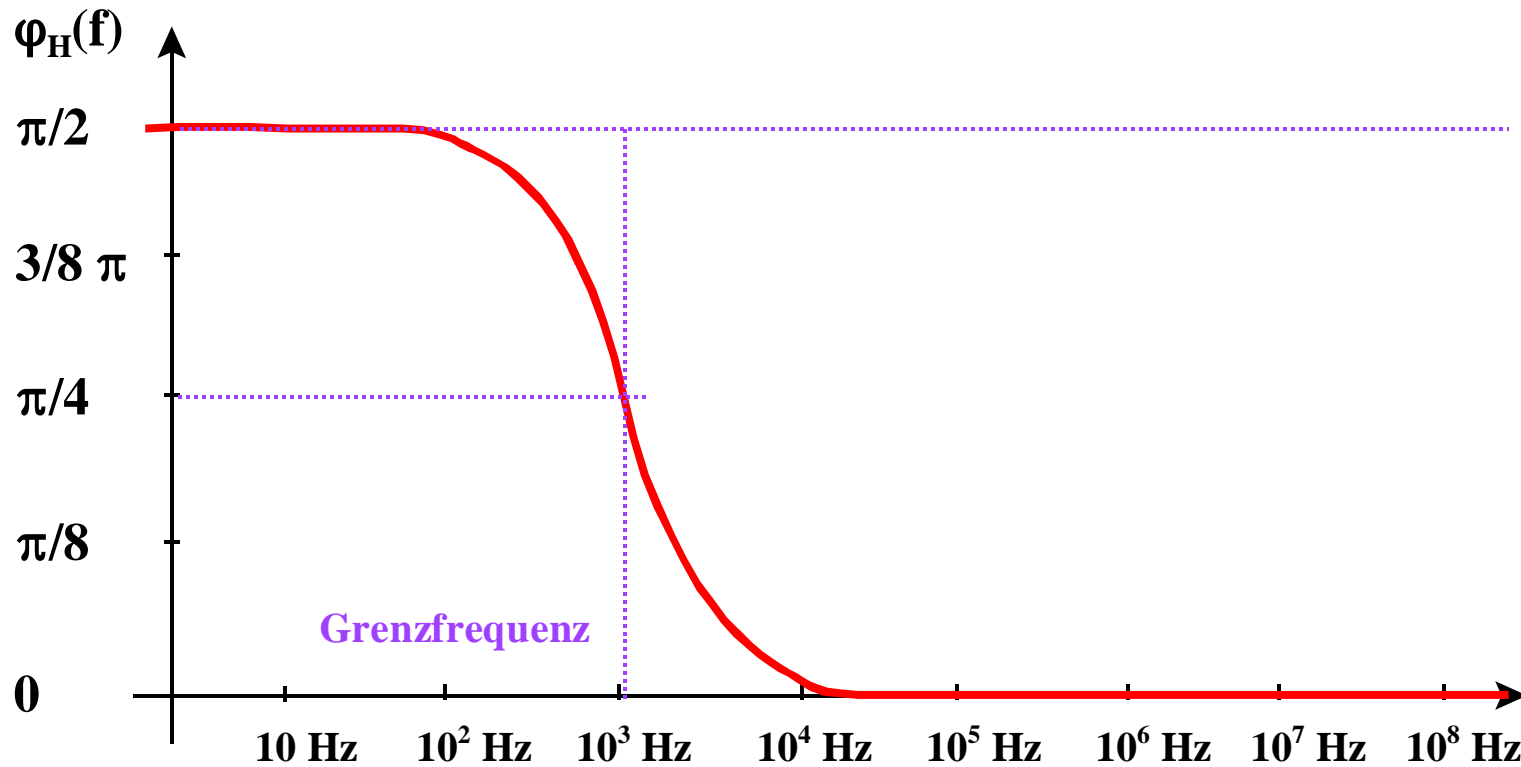
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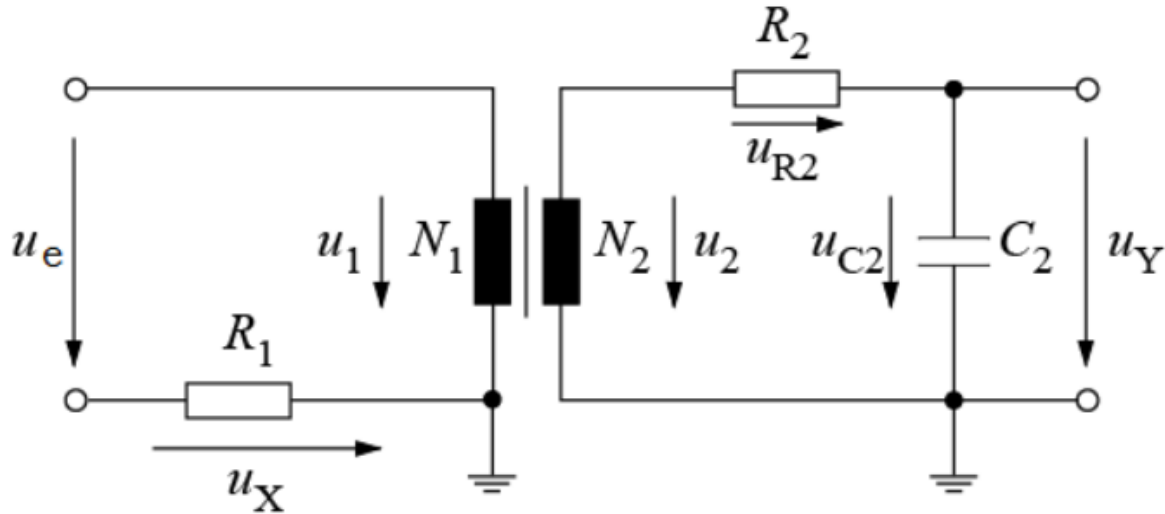
Grafische Darstellung des Phasengangs:

$$\varphi_H(\omega) = 0 + \arctan\left(\frac{R}{\omega L}\right)$$

$$\varphi_H(\omega) = \arctan\left(\frac{\omega_g}{\omega}\right) \quad \text{oder} \quad \varphi_H(f) = \arctan\left(\frac{f_g}{f}\right) \quad \omega_g = \frac{R}{L}$$



### c) RC-Tiefpass als Integrierglied



Messwiderstand  $R_1=100\Omega$ ,  $R_2=30k\Omega$ ,  $C_2=4,7\mu F$ ,  
 $N_1=2600$ ,  $N_2=90$ ,  $l_{Fe}=13cm$ ,  $A_{Fe}=3,1cm^2$

Aufnahme der B-H-Kennlinie des Eisenkerns, dazu Erfassung der Feldstärke  $H$  über Strom durch Primärwicklung  $N_1$ :

$$H = \frac{N_1 \cdot U_X}{l_{Fe} \cdot R_1}$$

B hat differenziellen Zusammenhang:

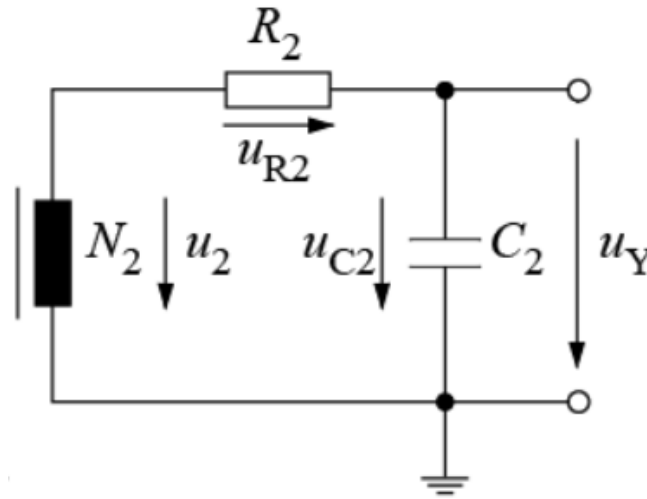
$$u_2 = \frac{N_2 \cdot d\Phi}{dt} = \frac{N_2 \cdot A_{Fe} \cdot dB}{dt}$$

$$dB = \frac{u_2(t)}{N_2 \cdot A_{Fe}} \cdot dt$$

$$B = \frac{1}{N_2 \cdot A_{Fe}} \int u_2(t) \cdot dt$$

$$dB = \frac{u_2(t)}{N_2 \cdot A_{Fe}} \cdot dt$$

$$B = \frac{1}{N_2 \cdot A_{Fe}} \int u_2(t) \cdot dt$$



$$u_2 = u_{R2} + u_{C2} \quad \text{wenn keine Last angeschlossen, gilt } i_{R2} = i_{C2} = i$$

$$u_2 = i \cdot R_2 + u_{C2} \quad \text{wenn } u_{R2} \gg u_{C2} \text{ (also } R_2 \gg \frac{1}{\omega \cdot C})$$

$$u_2 = i \cdot R_2 \quad i = C_2 \cdot \frac{du_{C2}}{dt}$$

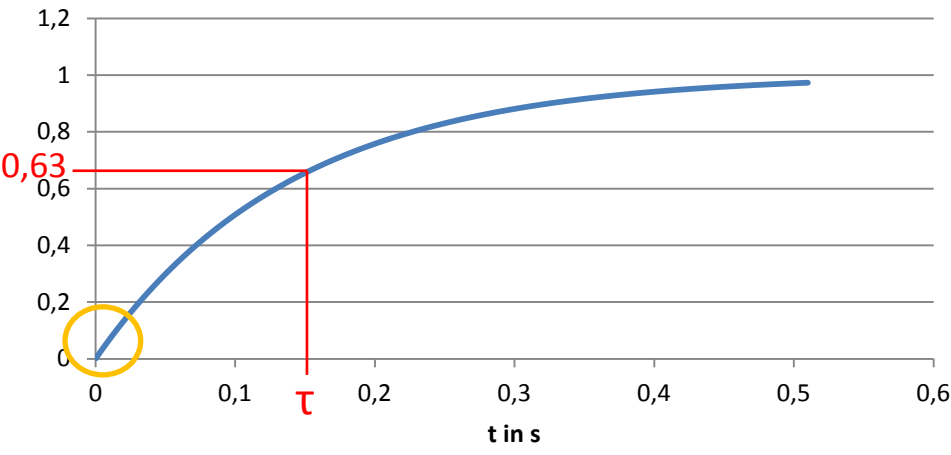
$$u_{R2} = R_2 \cdot C_2 \cdot \frac{du_{C2}}{dt} \quad du_{C2} = \frac{1}{R_2 \cdot C_2} u_{R2} \cdot dt$$

$$u_{C2} = \frac{1}{R_2 \cdot C_2} \int u_{R2} \cdot dt$$

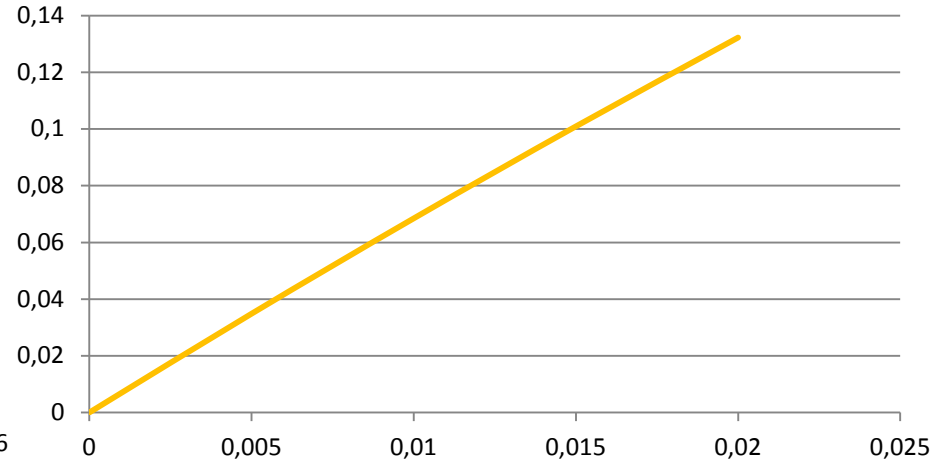


$$u_{C2} = \frac{1}{R_2 \cdot C_2} \int u_{R2} \cdot dt$$

Uc



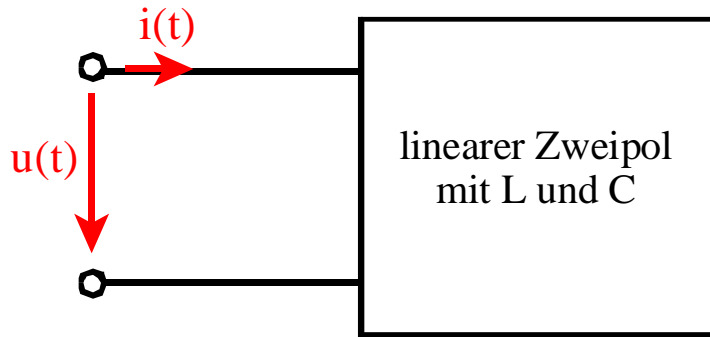
Uc



$$\tau = R_2 \cdot C_2 = 30k\Omega \cdot 4,7\mu F = 0,141s$$

Integrationszeit  $t_i$  in GET2:  
 $f=50 \text{ Hz} \Rightarrow T=1/f=0,02s$ ,  
 also  $T \ll \tau$  und  $u_c \ll u_R$

# Die Resonanzbedingungen



Ein linearer Zweipol befindet sich dann in Resonanz, wenn er sich trotz Vorhandensein von mindestens einer Induktivität und mindestens einer Kapazität bei Wechselstrom wie ein Ohmscher Widerstand verhält, wenn also nur die "verbrauchte" Wirkleistung von der äußeren Schaltung nachgeliefert werden muß.

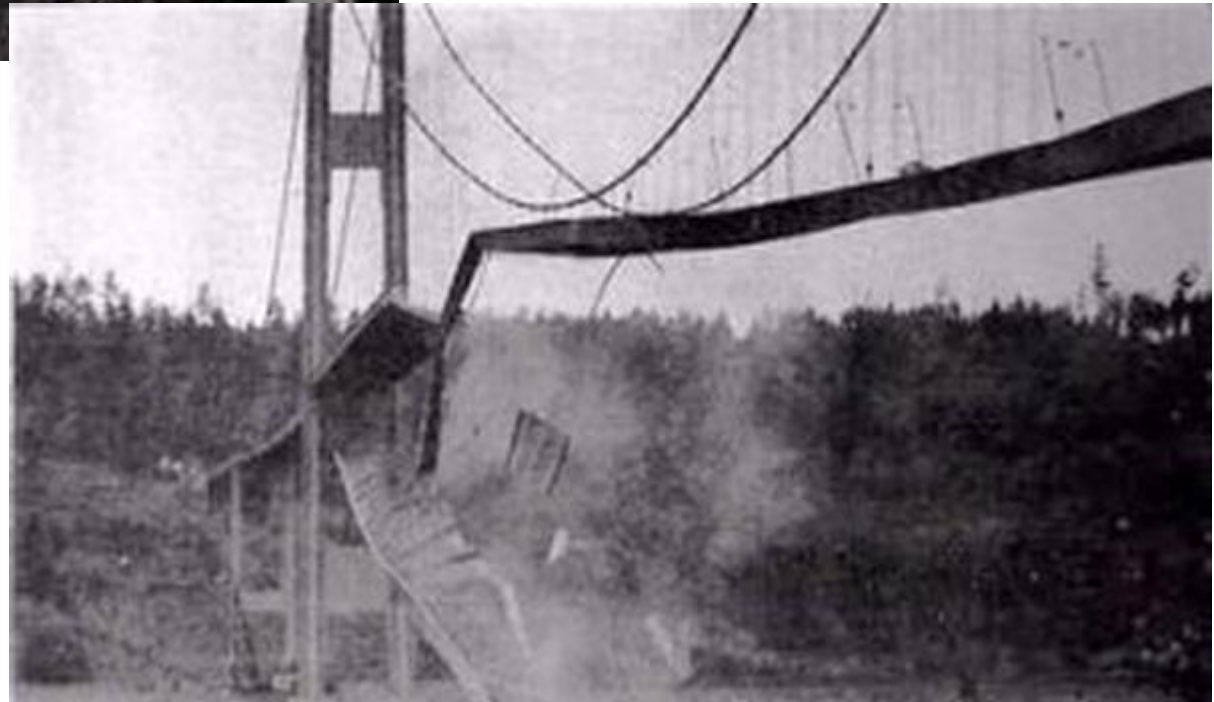
$$\operatorname{Im}\{\underline{Z}\} = 0$$

$$\operatorname{Im}\{\underline{Y}\} = 0$$

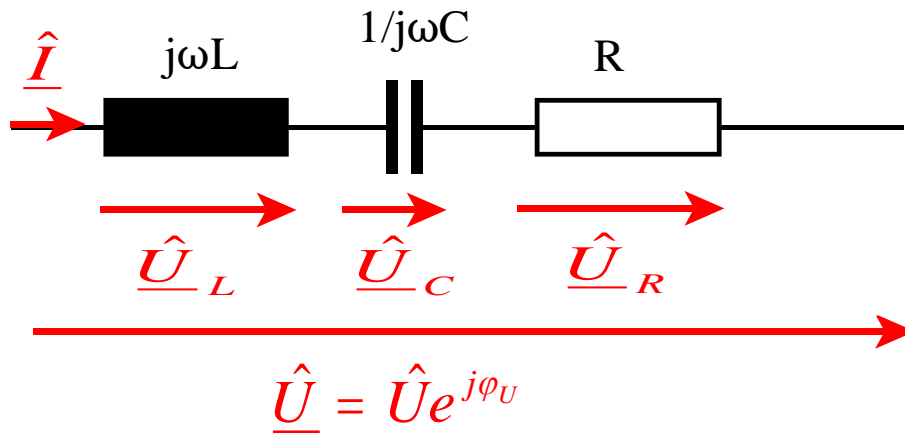
$$\varphi_{UI} = \varphi_U - \varphi_I = 0$$



Tacomabrücke



## Die Reihen- oder Spannungsresonanz



$$\underline{Z} = R + j\omega L + \frac{1}{j\omega C}$$

$$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Thomsonsche Schwingungsformel

Bestimmung der Resonanzfrequenz

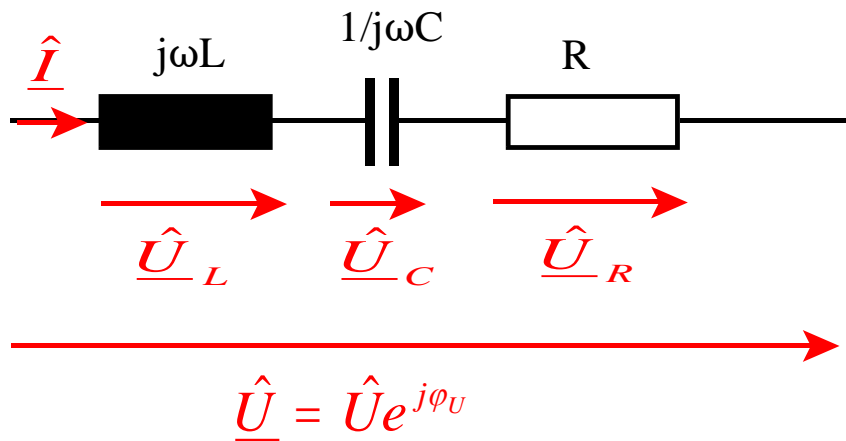
$$\text{Im}\{Z(\omega_0)\} = 0$$

$$\omega_0 L - \frac{1}{\omega_0 C} = 0$$

$$\omega_0^2 LC - 1 = 0$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$



$$\underline{Z} = R + j\omega L + \frac{1}{j\omega C}$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

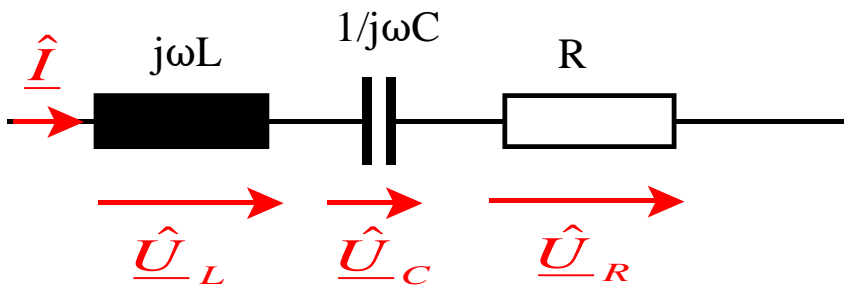
Die Abhängigkeit des Stromes von der Frequenz

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R + j\left[\frac{\omega}{\omega_0} \omega_0 L - \frac{1}{\omega_0} \frac{1}{\omega_0 C}\right]}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R + j\omega_0 L \left[\frac{\omega}{\omega_0} - \frac{1}{\omega}\right]}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R \left[1 + j \frac{\omega_0 L}{R} \left[\frac{\omega}{\omega_0} - \frac{1}{\omega}\right]\right]}$$



$$\omega_0 L = \frac{1}{\omega_0 C}$$

mit den Abkürzungen

$$\frac{\omega}{\omega_0} = \frac{f}{f_0} = x$$

normierte Frequenz

$$\underline{\hat{U}} = \hat{U} e^{j\varphi_U}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R \left[ 1 + j \frac{\omega_0 L}{R} \left( \frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}} \right) \right]}$$

$$\frac{\omega_0 L}{R} = Q_R$$

Güte

$$Q_R = \frac{\text{Leistung an Loder C}}{\text{Leistung an R}}$$

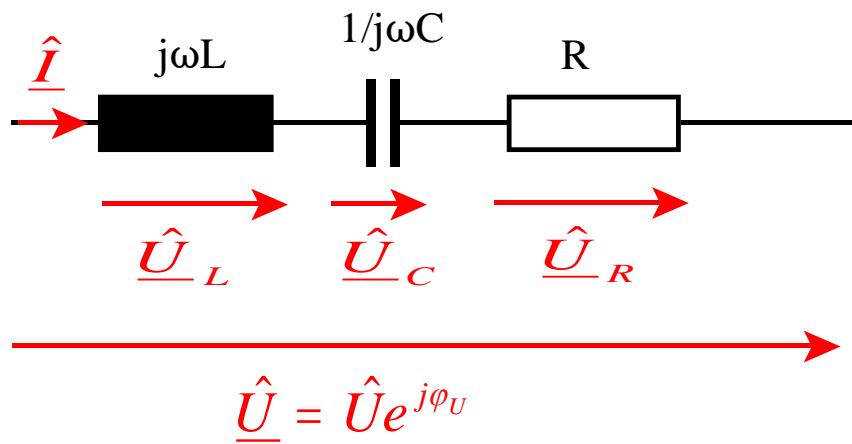
$$\underline{\hat{I}}_0 = \frac{\hat{U}}{R}$$

Strom im Resonanzfall

entsteht

$$\underline{\hat{I}} = \frac{\underline{\hat{I}}_0}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$

$$\frac{\underline{\hat{I}}}{\underline{\hat{I}}_0} = \frac{1}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$



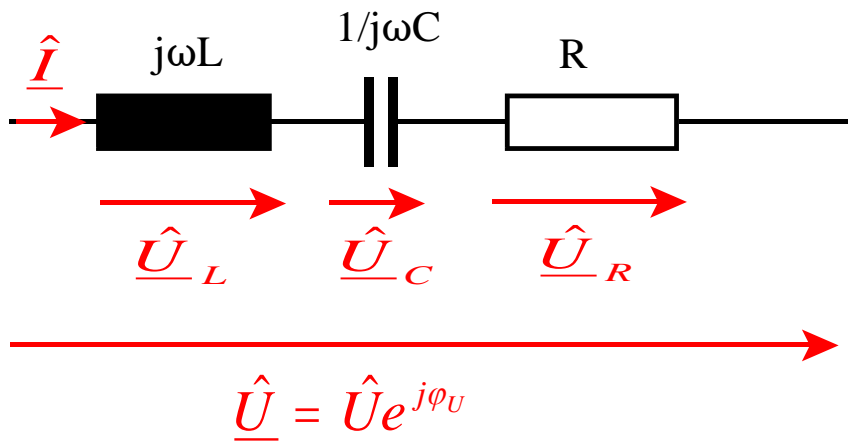
$$\frac{\underline{\hat{I}}}{\underline{\hat{I}}_0} = \frac{1}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$

$$\left| \frac{\underline{\hat{I}}}{\underline{\hat{I}}_0} \right| = \frac{1}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

Betrag des Stromes

$$\varphi_{I, I_0} = -\arctan \left( Q_R \left( x - \frac{1}{x} \right) \right)$$

Phasenlage des Stromes



Berechnung der Spannung am Widerstand  
in Abhängigkeit von der Frequenz

$$\underline{\hat{U}}_R = R \underline{\hat{I}}$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$

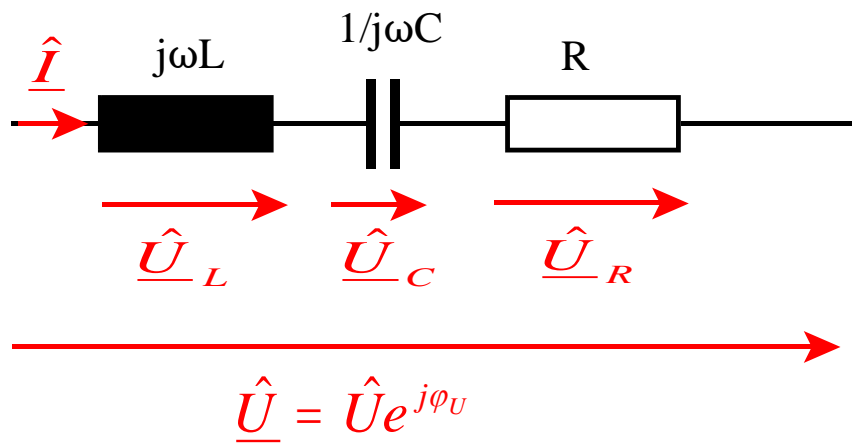
$$\underline{\hat{I}} = \frac{\underline{\hat{I}}_0}{1 + jQ_R \left( x - \frac{1}{x} \right)} \quad \underline{\hat{I}}_0 = \frac{\hat{U}}{R}$$

$$\underline{\hat{U}}_R = \frac{R \underline{\hat{U}}}{R \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$

$$\underline{\hat{U}}_R = \frac{\underline{\hat{U}}}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$

$$\frac{\underline{\hat{U}}_R}{\underline{\hat{U}}} = \frac{1}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$

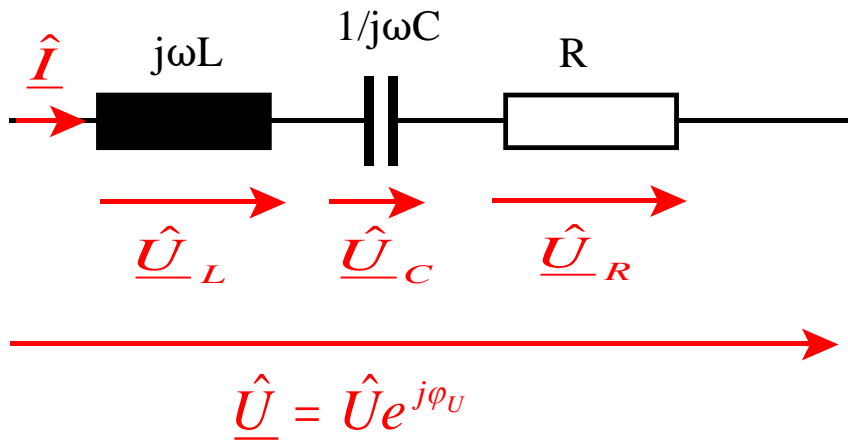




$$\frac{\hat{U}_R}{\hat{U}} = \frac{1}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$

$$\left| \frac{\hat{U}_R}{\hat{U}} \right| = \frac{1}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\varphi_{U_R, U} = -\arctan \left( Q_R \left( x - \frac{1}{x} \right) \right)$$



Berechnung der Spannung an der Induktivität  
in Abhängigkeit von der Frequenz

$$\underline{\hat{U}}_L = j\omega L \underline{\hat{I}} = j \frac{\omega}{\omega_0} \omega_0 L \underline{\hat{I}}$$

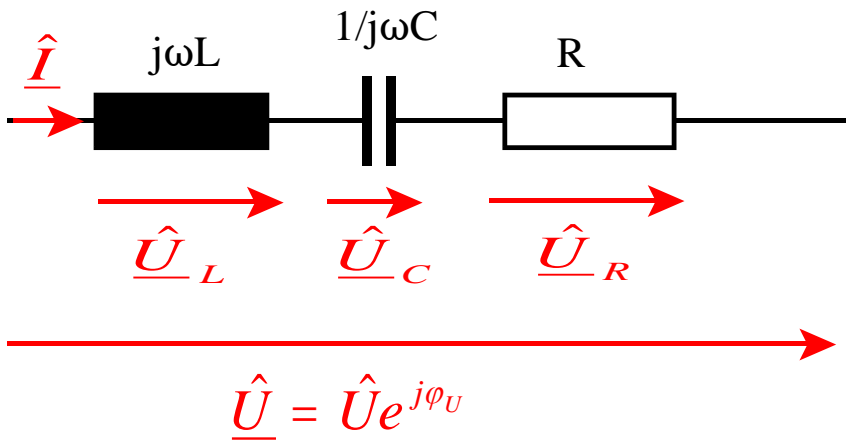
$$\underline{\hat{U}}_L = \frac{xj\omega_0 L \underline{\hat{U}}}{R \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$

$$\frac{\omega_0 L}{R} = Q_R$$

$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$

$$\underline{\hat{U}}_L = \frac{xjQ_R \underline{\hat{U}}}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$

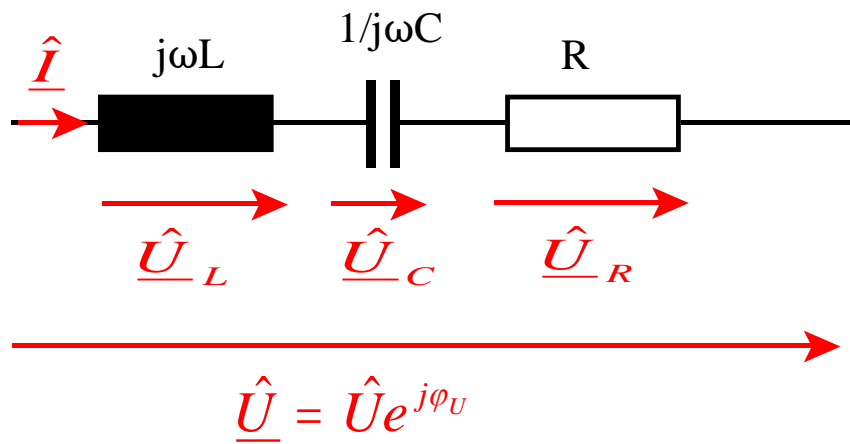
$$\frac{\underline{\hat{U}}_L}{\underline{\hat{U}}} = \frac{xjQ_R}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$



$$\frac{\hat{U}_L}{\hat{U}} = \frac{xjQ_R}{1 + jQ_R \left( x - \frac{1}{x} \right)}$$

$$\left| \frac{\hat{U}_L}{\hat{U}} \right| = \frac{xQ_R}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\varphi_{U_L, U} = \frac{\pi}{2} - \arctan \left( Q_R \left( x - \frac{1}{x} \right) \right)$$



$$\underline{\hat{I}} = \frac{\underline{\hat{U}}}{R \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$

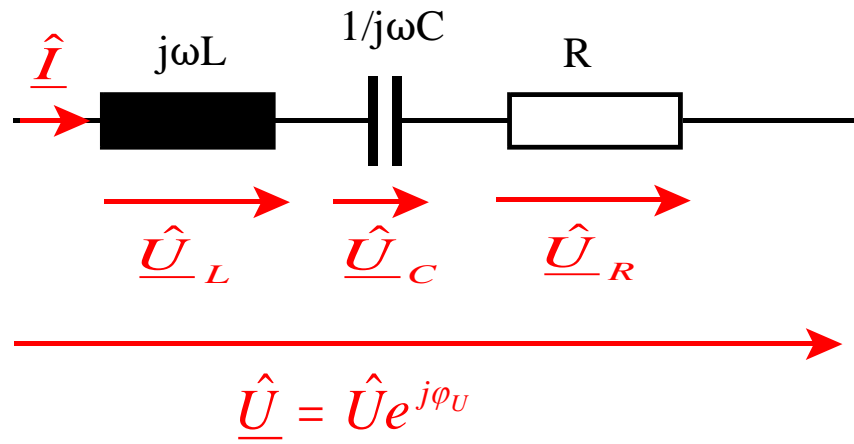
Berechnung der Spannung an der Kapazität in Abhängigkeit von der Frequenz

$$\underline{\hat{U}}_C = \frac{1}{j\omega C} \underline{\hat{I}} = \frac{\omega_0}{\omega} \frac{1}{j\omega_0 C} \underline{\hat{I}}$$

$$\underline{\hat{U}}_C = \frac{\underline{\hat{U}}}{x j \omega_0 C R \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$

$$\underline{\hat{U}}_C = \frac{-jQ_R \underline{\hat{U}}}{x \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$

$$\frac{\underline{\hat{U}}_C}{\underline{\hat{U}}} = \frac{-jQ_R}{x \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$



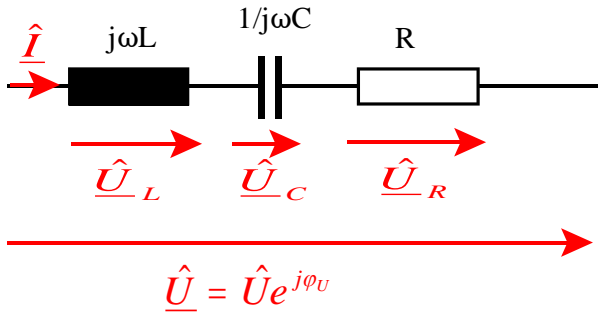
$$\frac{\underline{\hat{U}}_C}{\underline{\hat{U}}} = \frac{-jQ_R}{x \left[ 1 + jQ_R \left( x - \frac{1}{x} \right) \right]}$$

$$\left| \frac{\underline{\hat{U}}_C}{\underline{\hat{U}}} \right| = \frac{Q_R}{x \sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\varphi_{U_C, U} = -\frac{\pi}{2} - \arctan \left( Q_R \left( x - \frac{1}{x} \right) \right)$$

# Die Resonanzkurven:

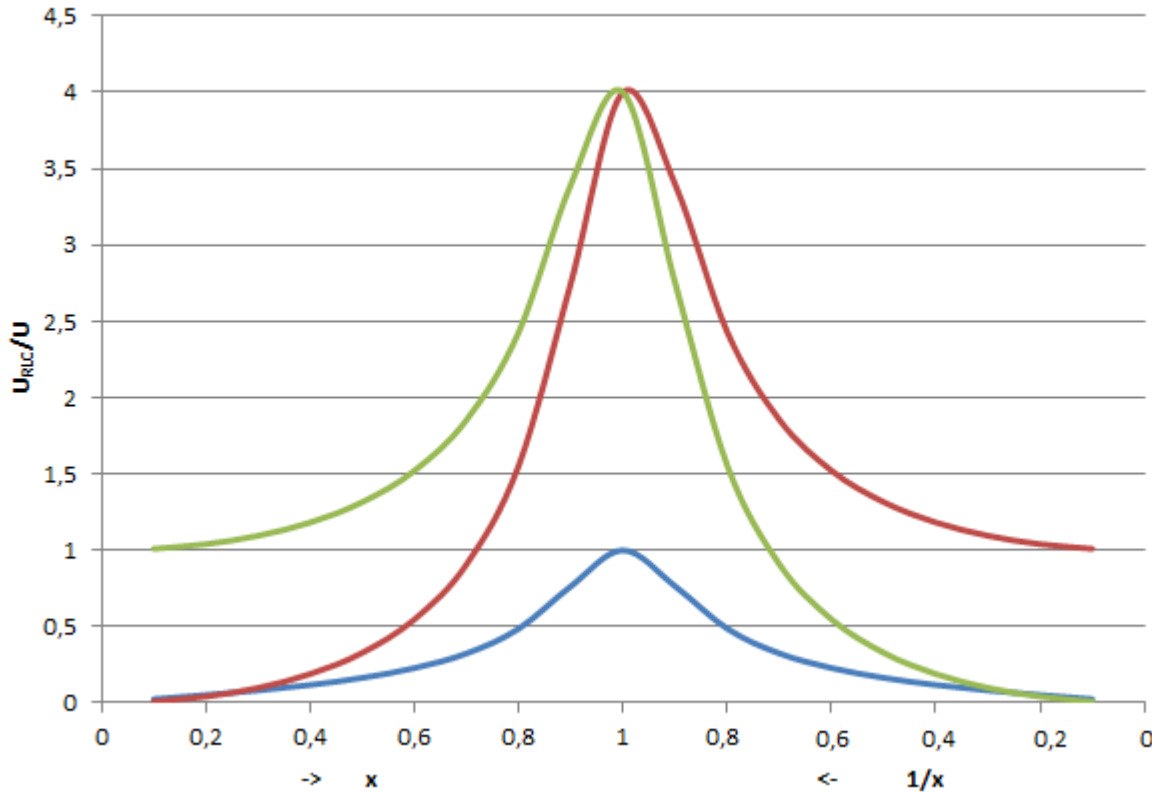
# Parameter des Schwingkreises:



$$L = 0.0001\text{H} \quad C = 1.0\mu\text{F} \quad R = 2.5 \Omega$$

$$f_0 = \frac{1}{2 \cdot \pi \cdot \sqrt{LC}} = 15,9\text{kHz}$$

$$Q_R = \frac{2\pi f_0 L}{R} = 4$$



$$\left| \frac{\hat{U}_R}{\hat{U}} \right| = \frac{1}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\left| \frac{\hat{U}_L}{\hat{U}} \right| = \frac{x Q_R}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\left| \frac{\hat{U}_C}{\hat{U}} \right| = \frac{Q_R}{x \sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

Besonderheiten im Resonanzfall:

$$x = 1 \text{ und } x - \frac{1}{x} = 0$$

$$\left| \frac{\hat{U}_R}{\hat{U}} \right| = \frac{1}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

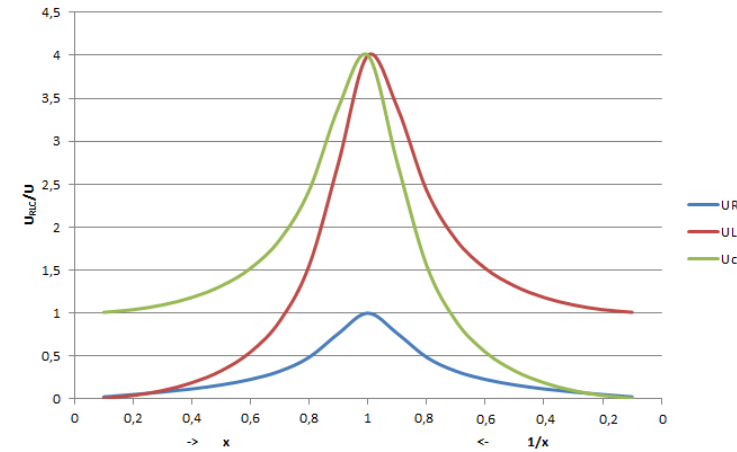
$$\hat{U}_{R0} = \hat{U}$$

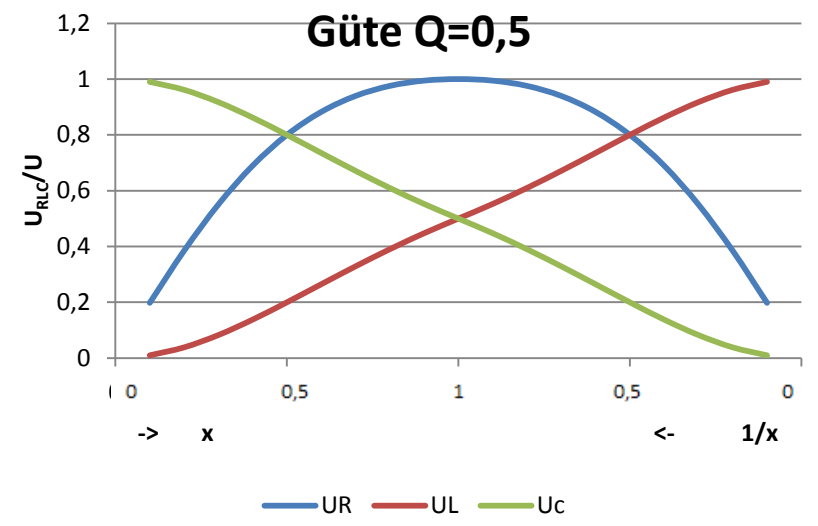
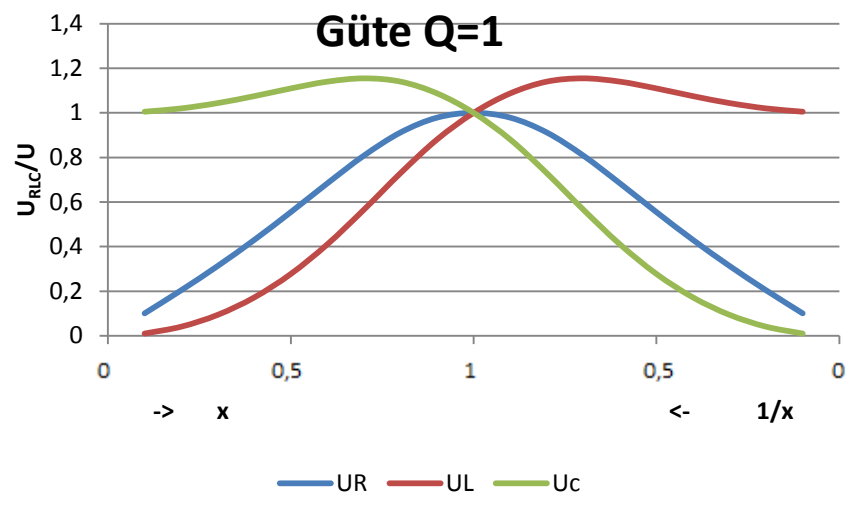
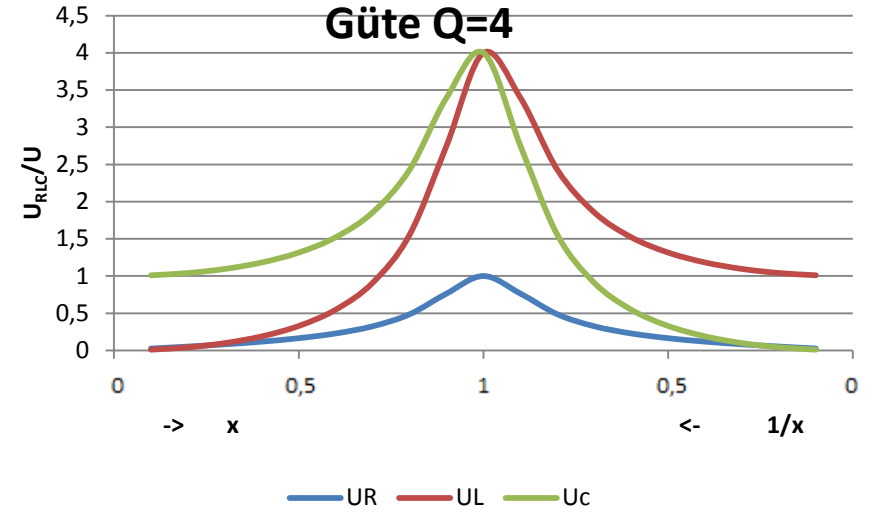
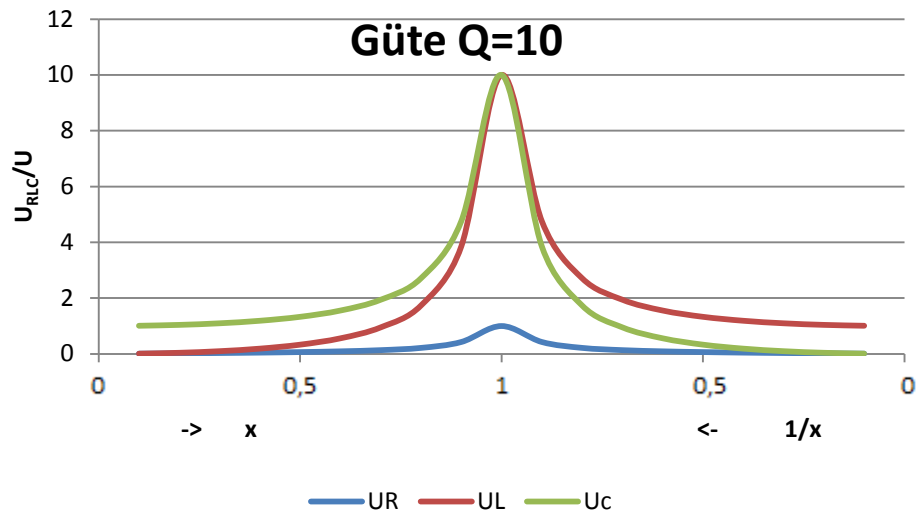
$$\left| \frac{\hat{U}_L}{\hat{U}} \right| = \frac{x Q_R}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\hat{U}_{L0} = Q_R \cdot \hat{U} \text{ (Spannungsüberhöhung)}$$

$$\left| \frac{\hat{U}_C}{\hat{U}} \right| = \frac{Q_R}{x \sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

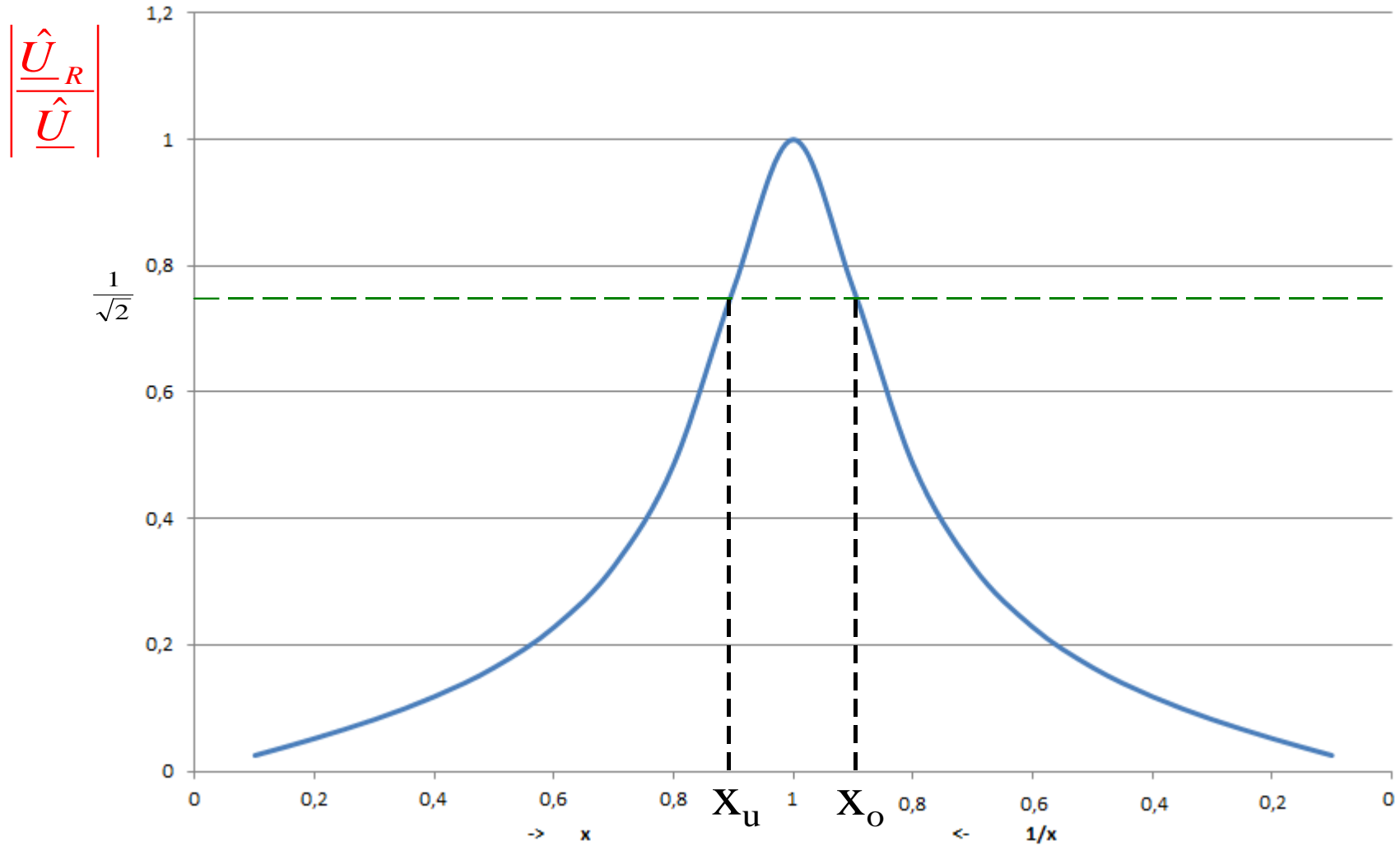
$$\hat{U}_{C0} = Q_R \cdot \hat{U} \text{ (Spannungsüberhöhung)}$$

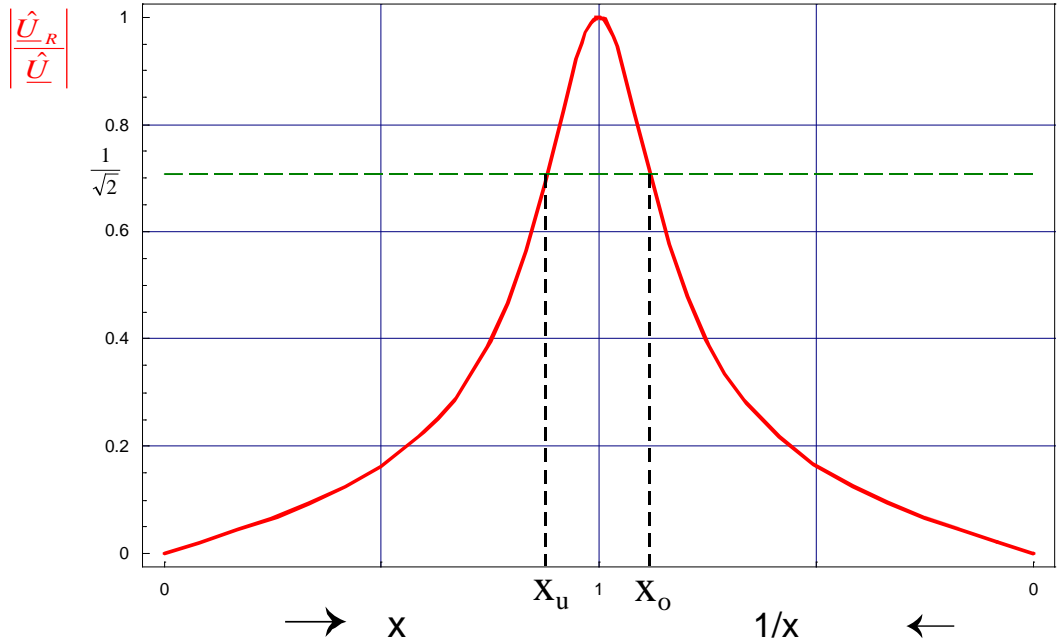






- der Zusammenhang zwischen Bandbreite und Güte





$$\left| \frac{\hat{U}_R}{\hat{U}} \right| = \frac{1}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$f_g$  oder  $\omega_g$ :  
 Im ( $|H(\omega)| = \text{Re}(|H(\omega)|$ )  
 (auch 45°-Eckfrequenz genannt)

$$Q_R^2 \left( x_{o,u} - \frac{1}{x_{o,u}} \right)^2 = 1$$

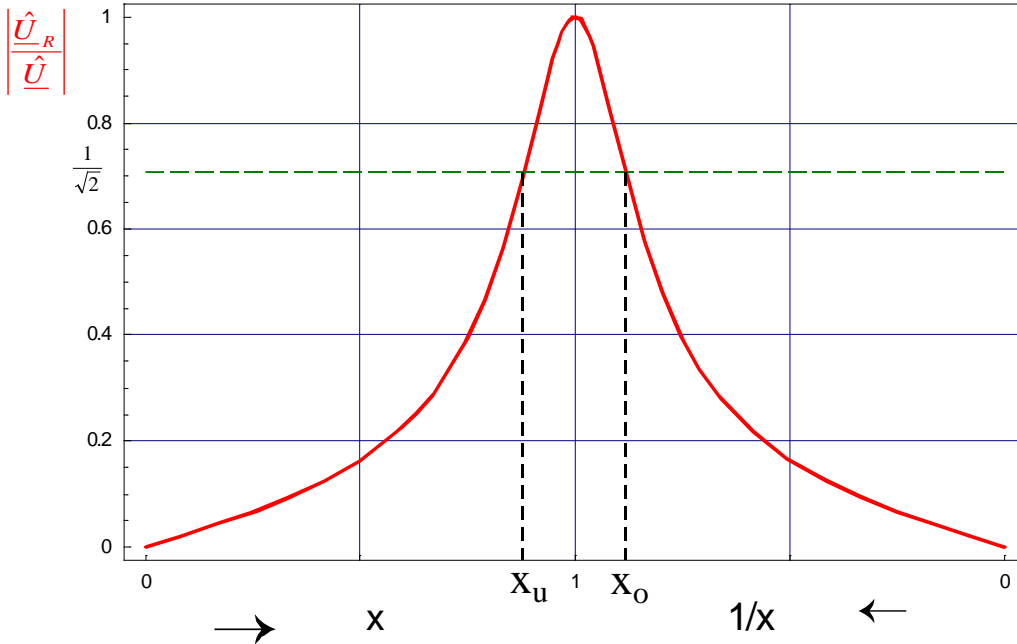
$$Q_R \left( x_o - \frac{1}{x_o} \right) = 1 \quad Q_R \left( x_u - \frac{1}{x_u} \right) = -1$$

$$x_o = \frac{f_{ob}}{f_0} \quad x_u = \frac{f_u}{f_0}$$

$$\frac{f_{ob}}{f_0} - \frac{f_0}{f_{ob}} = \frac{1}{Q_R}$$

$$\frac{f_{ob}^2 - f_0^2}{f_0 f_{ob}} = \frac{1}{Q_R}$$

$$\frac{(f_{ob} + f_0)(f_{ob} - f_0)}{f_0 f_{ob}} = \frac{1}{Q_R}$$



$$\left| \frac{\hat{U}_R}{\hat{U}} \right| = \frac{1}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\frac{(f_{ob} + f_0)(f_{ob} - f_0)}{f_0 f_{ob}} = \frac{1}{Q_R}$$

$$x_o = \frac{f_{ob}}{f_0} \quad x_u = \frac{f_u}{f_0}$$

$$Q_R \left( x_u - \frac{1}{x_u} \right) = -1$$

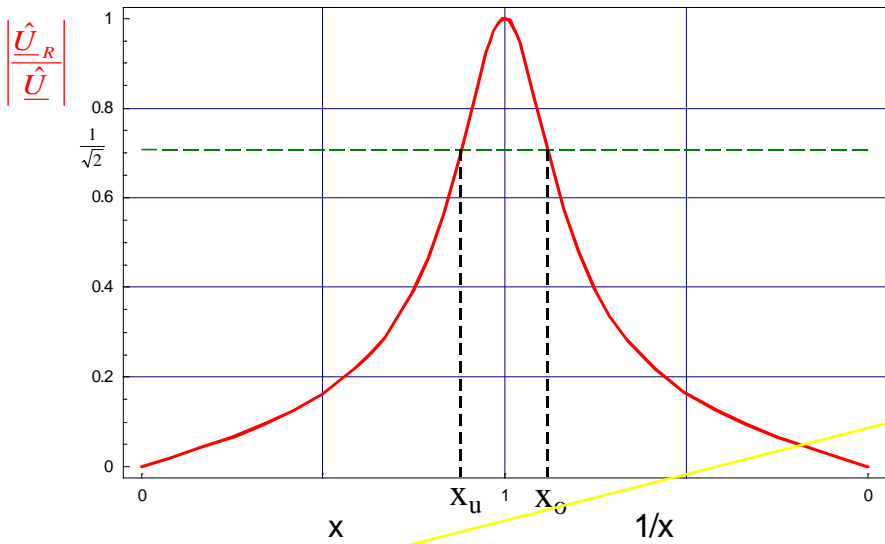
bei großer Güte gilt in guter Näherung

$$f_{ob} \approx f_u \approx f_0$$

$$\frac{(f_u + f_0)(f_u - f_0)}{f_0 f_u} = -\frac{1}{Q_R}$$

und für die Bandbreite  $b$

$$b = f_{ob} - f_u$$



$$\left| \frac{\hat{U}_R}{\hat{U}} \right| = \frac{1}{\sqrt{1 + Q_R^2 \left( x - \frac{1}{x} \right)^2}}$$

$$f_{ob} \approx f_u \approx f_0$$

$$b = f_{ob} - f_u$$

$$f_{ob} = \left( \frac{1}{2Q_R} + 1 \right) f_0$$

$$f_u = \left( -\frac{1}{2Q_R} + 1 \right) f_0$$

$$b = \left[ \frac{1}{2 \cdot Q_R} + 1 - \left( -\frac{1}{2 \cdot Q_R} + 1 \right) \right] \cdot f_0$$

$$\frac{(f_{ob} + f_0)(f_{ob} - f_0)}{f_0 f_{ob}} = \frac{1}{Q_R}$$

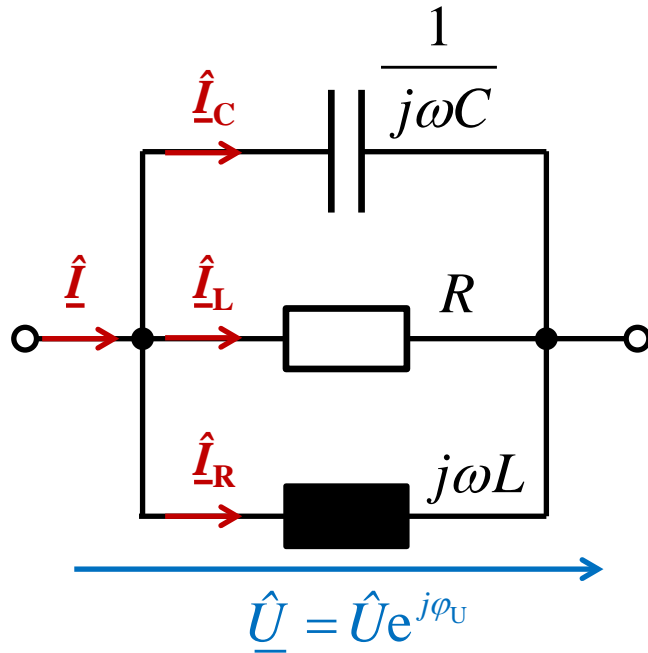
$$\frac{2 \cdot f_{ob} \cdot (f_{ob} - f_0)}{f_0 \cdot f_{ob}} = \frac{1}{Q_R} \quad \frac{2(f_{ob} - f_0)}{f_0} = \frac{1}{Q_R}$$

$$\frac{(f_u + f_0)(f_u - f_0)}{f_0 f_u} = -\frac{1}{Q_R} \quad \frac{2(f_u - f_0)}{f_0} = -\frac{1}{Q_R}$$

$$b = \frac{1}{Q_R} f_0$$

## b) die Parallel- oder Stromresonanz

(der ideale Parallelschwingkreis)



Eingeprägter Strom!

Beispiel:  
 $R=5\text{k}\Omega$   
 $L=1\text{H}$   
 $C=1\mu\text{F}$   
 $\hat{I}=10\text{mA}$

- Bestimmung der Resonanzfrequenz

$$\text{Im}\{Y(\omega_0)\} = 0$$

$$\omega_0 C - \frac{1}{\omega_0 L} = 0$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\underline{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

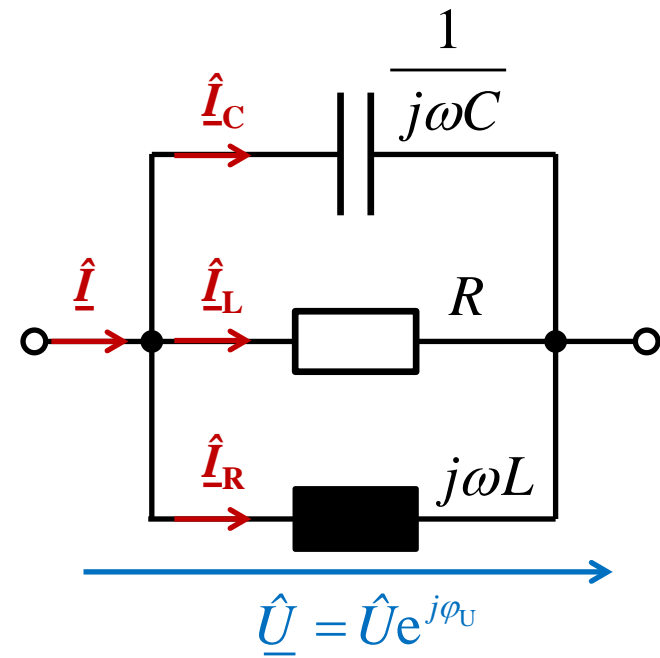
Resonanzfrequenz im  
Beispielschwingkreis:  
 $f_0=225,08\text{ Hz}$

- die Abhängigkeit der Spannung  $\underline{\hat{U}}$  von der Kreisfrequenz

$$\underline{\hat{U}}(\omega) = \frac{\underline{\hat{I}}}{\underline{Y}} = \frac{\underline{\hat{I}}}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

mit der Resonanzkreisfrequenz folgt:

$$\underline{\hat{U}}(\omega) = \frac{\underline{\hat{I}}}{\frac{1}{R} + j\left(\frac{\omega}{\omega_0} \omega_0 C - \frac{1}{\frac{\omega}{\omega_0} \omega_0 L}\right)}$$



$$\underline{Y} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$

$$\omega_0 C = \frac{1}{\omega_0 L}$$

$$\underline{\hat{U}}(\omega) = \frac{\underline{\hat{I}}}{\frac{1}{R} + j\omega_0 C \left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right)} = \frac{\underline{\hat{I}}}{\frac{1}{R} \left[ 1 + j\omega_0 C R \left(\frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}}\right) \right]}$$

$$\underline{\hat{U}} = \frac{\underline{\hat{I}}}{\frac{1}{R} \left[ 1 + j\omega_0 CR \left( \frac{\omega}{\omega_0} - \frac{1}{\omega} \right) \right]}$$

Mit den Abkürzungen

normierte Frequenz, Güte und Spannung bei Resonanz

$$x = \frac{\omega}{\omega_0} = \frac{f}{f_0}$$

Bei Resonanz gilt:

$$Q_P = \frac{\text{Leistung an Loder C}}{\text{Leistung an R}}$$

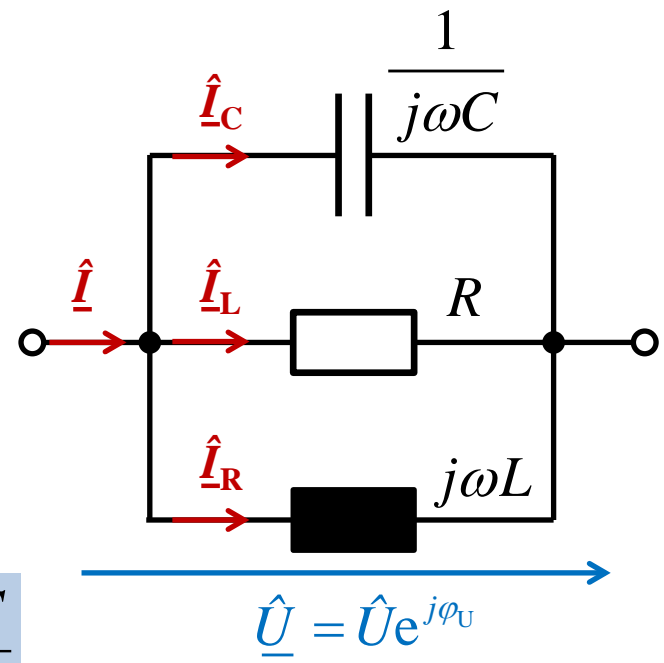
$$Q_P = \frac{U_0^2 / \omega_0 L}{U_0^2 / R} \text{ oder } \frac{U_0^2 * \omega_0 C}{U_0^2 / R}$$

$$Q_P = \omega_0 CR = \frac{R}{\omega_0 L}$$

Mit den Beispielwerten wird  $Q_p=3,536$

$$\underline{\hat{U}}_0 = \underline{\hat{I}}R$$

Mit den Beispielwerten wird  $\hat{U}_0 = 10mA * 5k\Omega = 50V$



Beispiel:  
 $R=5k\Omega$   
 $L=1H$   
 $C=1\mu F$   
 $\hat{I}=10mA$

$$\underline{\hat{U}} = \frac{\underline{\hat{I}}}{\frac{1}{R} \left[ 1 + j\omega_0 CR \left( \frac{\omega}{\omega_0} - \frac{1}{\frac{\omega}{\omega_0}} \right) \right]}$$

$$x = \frac{\omega}{\omega_0} = \frac{f}{f_0} \quad Q_P = \omega_0 CR = \frac{R}{\omega_0 L} \quad \underline{\hat{U}}_0 = \underline{\hat{I}}R$$

$$\underline{\hat{U}} = \frac{\underline{\hat{U}}_0}{1 + jQ_P \left( x - \frac{1}{x} \right)} \quad \text{bzw.} \quad \frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} = \frac{1}{1 + jQ_P \left( x - \frac{1}{x} \right)}$$

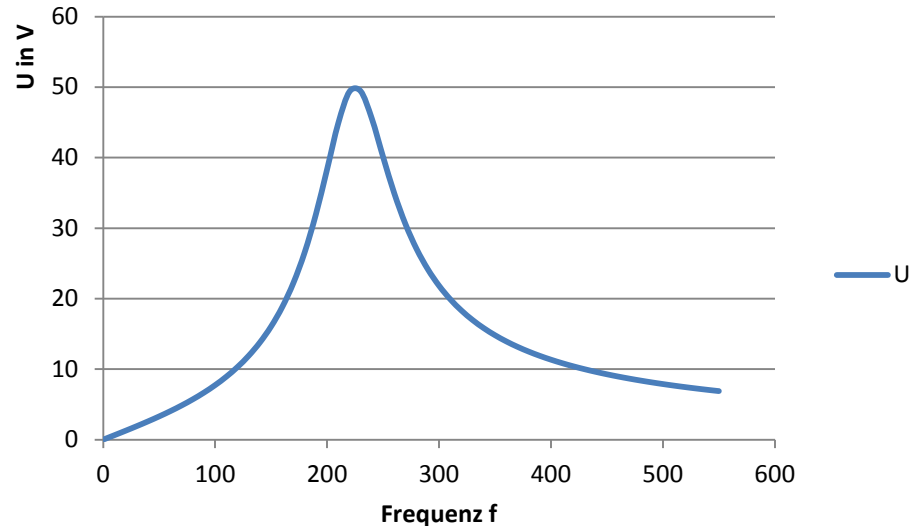
Aus obigen Beziehungen folgt:

mit Betrags- und Phasenverlauf:

$$\left| \frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} \right| = \frac{1}{\sqrt{1 + Q_P^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\varphi_U = -\arctan \left( Q_P \left( x - \frac{1}{x} \right) \right)$$

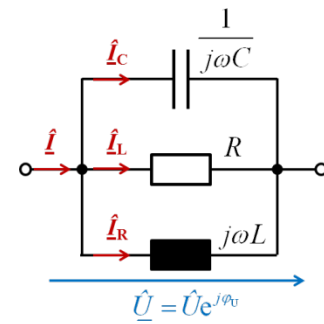
Mit den Beispielwerten ergibt sich für U:





$$\frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} = \frac{1}{1 + jQ_P \left( x - \frac{1}{x} \right)}$$

$$\underline{\hat{U}}_0 = \underline{\hat{I}}R$$



- die Berechnung des **Stromes durch den Widerstand** in Abhängigkeit von der norm. Frequenz

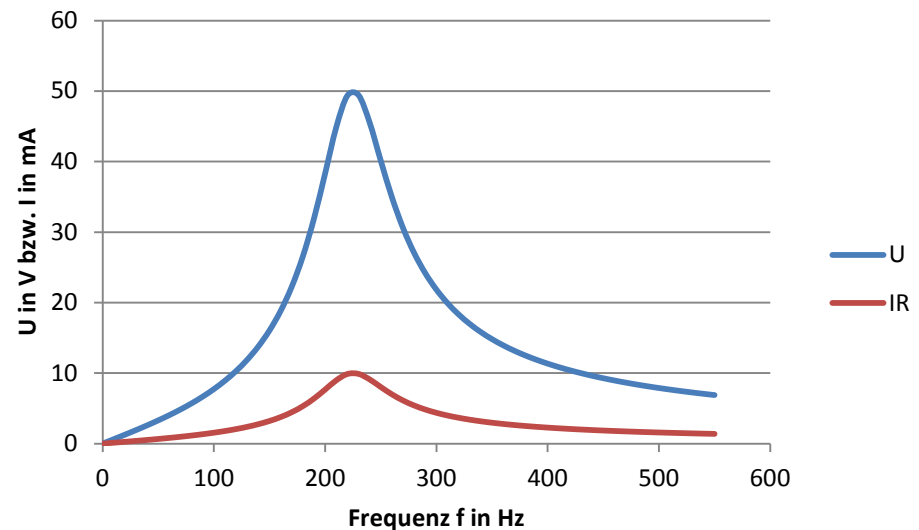
$$\left. \begin{aligned} \underline{\hat{I}}_R &= \frac{\underline{\hat{U}}}{R} \\ \underline{\hat{U}} &= \frac{\underline{\hat{I}}}{\frac{1}{R} \left[ 1 + jQ_P \left( x - \frac{1}{x} \right) \right]} \end{aligned} \right\}$$

$$\underline{\hat{I}}_R = \frac{1}{R} \frac{\underline{\hat{I}}}{\frac{1}{R} \left[ 1 + jQ_P \left( x - \frac{1}{x} \right) \right]} = \frac{\underline{\hat{I}}}{1 + jQ_P \left( x - \frac{1}{x} \right)}$$

$$\frac{\underline{\hat{I}}_R}{\underline{\hat{I}}} = \frac{1}{1 + jQ_P \left( x - \frac{1}{x} \right)}$$

mit Betrags- und Phasenverlauf:

$$\left| \frac{\underline{\hat{I}}_R}{\underline{\hat{I}}} \right| = \frac{1}{\sqrt{1 + Q_P^2 \left( x - \frac{1}{x} \right)^2}}$$

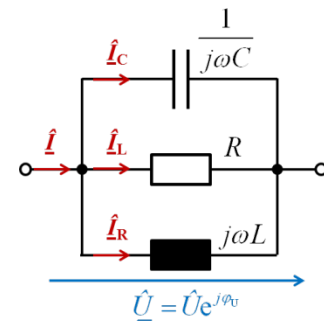


$$\varphi_{I_R, I} = -\arctan \left( Q_P \left( x - \frac{1}{x} \right) \right)$$

$$\frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} = \frac{1}{1 + jQ_P \left( x - \frac{1}{x} \right)}$$

$$\underline{\hat{U}}_0 = \underline{\hat{I}}R$$

$$Q_P = \omega_0 CR = \frac{R}{\omega_0 L}$$



- die Berechnung des **Stromes durch die Induktivität** in Abhängigkeit von der norm. Frequenz

$$\underline{\hat{I}}_L = \frac{\underline{\hat{U}}}{j\omega L} = \frac{\omega_0}{\omega} \frac{\underline{\hat{U}}}{j\omega_0 L}$$

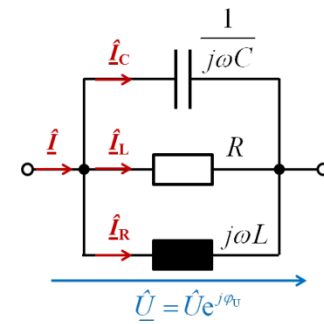
mit Betrags- und Phasenverlauf:

$$\left| \frac{\underline{\hat{I}}_L}{\underline{\hat{I}}} \right| = \frac{Q_P}{x \sqrt{1 + Q_P^2 \left( x - \frac{1}{x} \right)^2}} \quad \varphi_{I_L, I} = -\frac{\pi}{2} - \arctan \left( Q_P \left( x - \frac{1}{x} \right) \right)$$

$$\frac{\underline{\hat{U}}}{\underline{\hat{U}}_0} = \frac{1}{1 + jQ_P \left( x - \frac{1}{x} \right)}$$

$$\underline{\hat{U}}_0 = \underline{\hat{I}}R$$

$$Q_P = \omega_0 CR = \frac{\omega_0 L}{R}$$



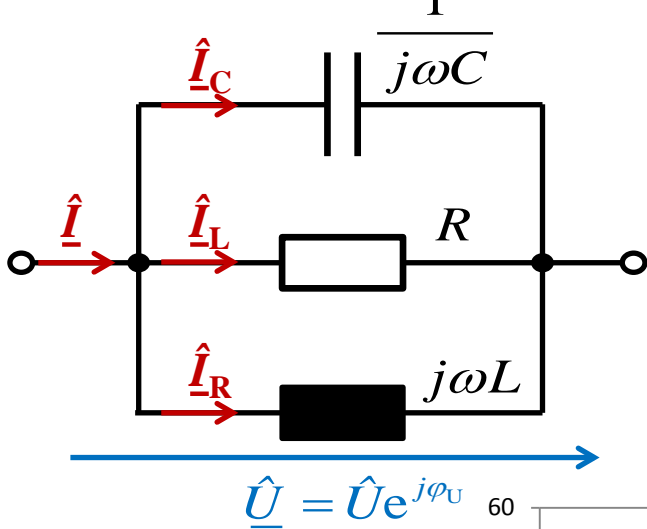
- die Berechnung des **Stromes durch die Kapazität** in Abhängigkeit von der norm. Frequenz

$$\underline{\hat{I}}_C = j\omega C \underline{\hat{U}} = \frac{\omega}{\omega_0} j\omega_0 C \underline{\hat{U}}$$

mit Betrags- und Phasenverlauf:

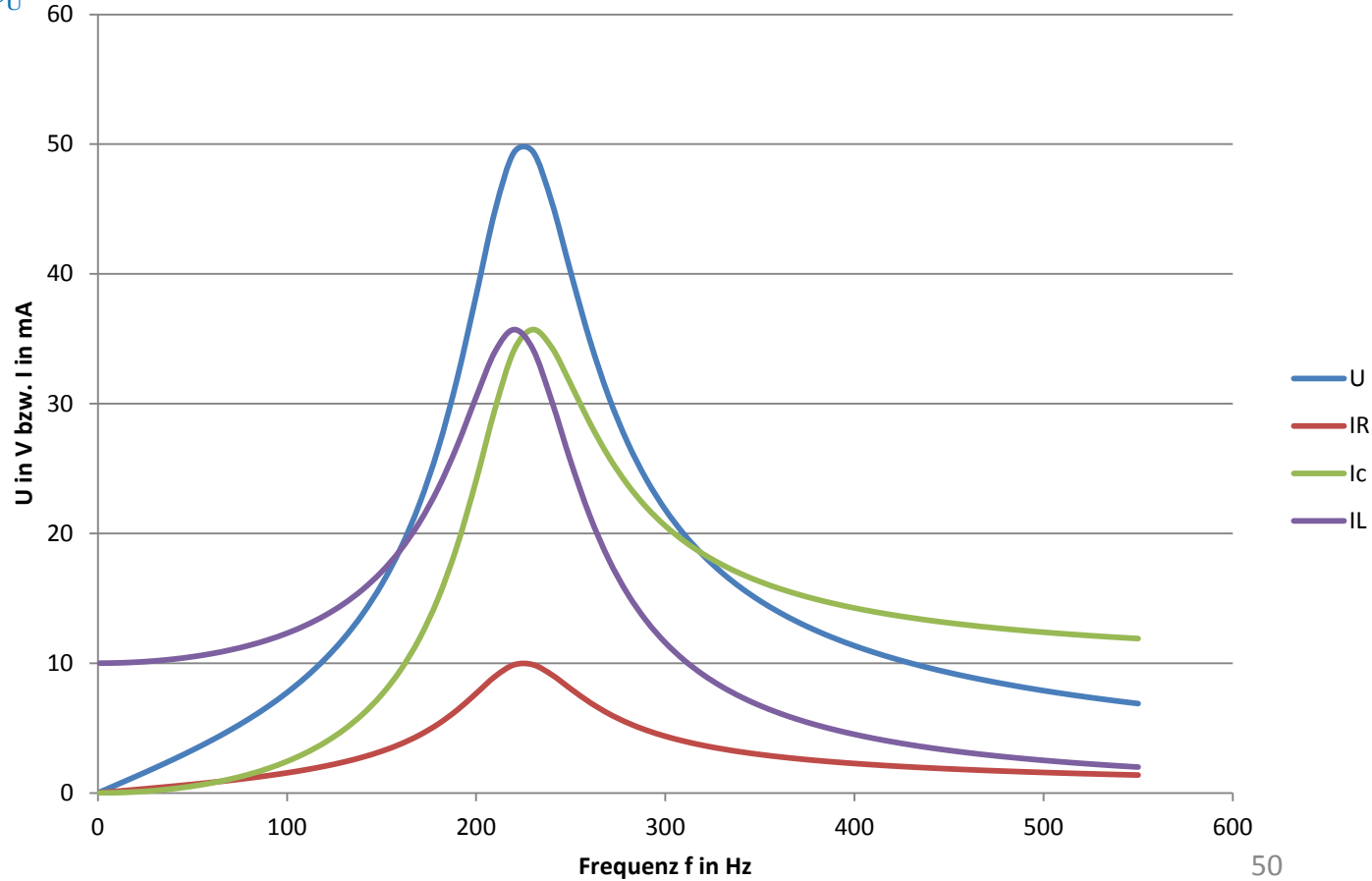
$$\left| \frac{\underline{\hat{I}}_C}{\underline{\hat{I}}} \right| = \frac{x Q_P}{\sqrt{1 + Q_P^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\varphi_{I_C, I} = \frac{\pi}{2} - \arctan \left( Q_P \left( x - \frac{1}{x} \right) \right)$$

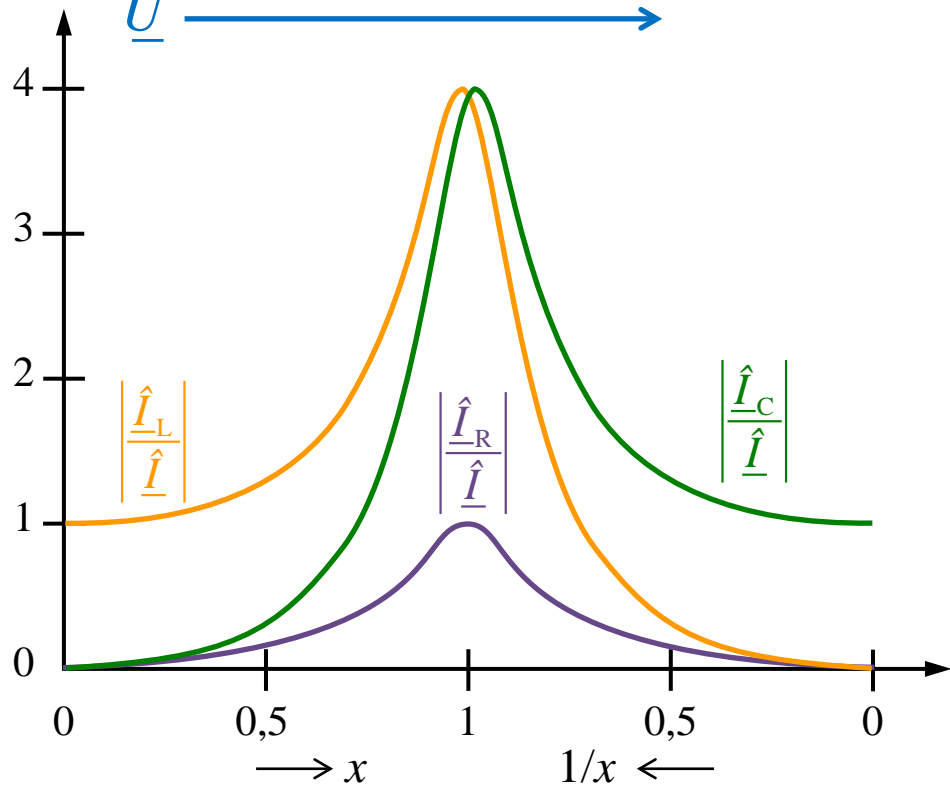
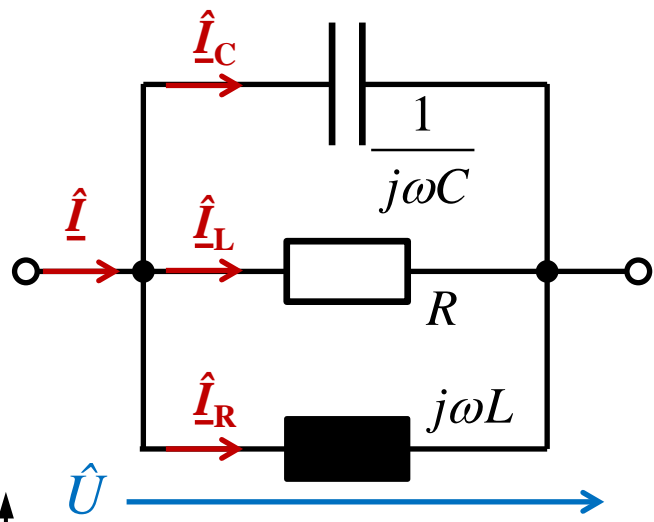


Beispiel:  
 $R=5\text{k}\Omega$   
 $L=1\text{H}$   
 $C=1\mu\text{F}$   
 $\hat{I}=10\text{mA}$

$f_0=225,08\text{ Hz}$   
 $Q_p=3,536$   
 $\hat{I}_{L0}=\hat{I}_{C0}=35,36\text{ mA}$



• die Resonanzkurven



Parameter des Schwingkreises:

$$L = 0,1 \text{ mH} \quad C = 1 \text{ } \mu\text{F} \quad R = 2,5 \text{ } \Omega$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \approx 15,9 \text{ kHz}$$

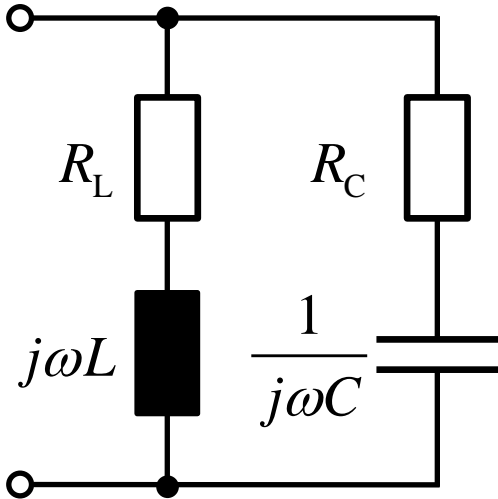
$$Q_P = \frac{2\pi f_0 C}{R} = 4$$

$$\left| \frac{\hat{I}_R}{\hat{I}} \right| = \frac{1}{\sqrt{1 + Q_P^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\left| \frac{\hat{I}_C}{\hat{I}} \right| = \frac{x Q_P}{\sqrt{1 + Q_P^2 \left( x - \frac{1}{x} \right)^2}}$$

$$\left| \frac{\hat{I}_L}{\hat{I}} \right| = \frac{Q_P}{x \sqrt{1 + Q_P^2 \left( x - \frac{1}{x} \right)^2}}$$

b) der reale Parallelschwingkreis



$$\underline{Y} = \frac{1}{R_L + j\omega L} + \frac{1}{R_C + \frac{1}{j\omega C}}$$

$$\underline{Y} = \frac{1}{R_L + j\omega L} + \frac{j\omega C}{j\omega C R_C + 1}$$

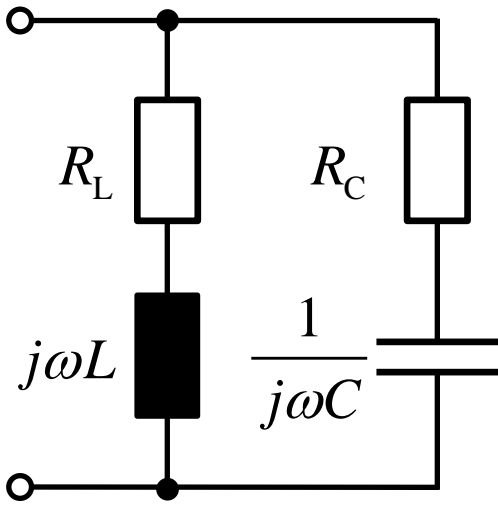
$$\underline{Y} = \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{j\omega C(-j\omega C R_C + 1)}{(\omega C R_C)^2 + 1}$$

$$\underline{Y} = \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + \frac{(\omega C)^2 R_C + j\omega C}{(\omega C R_C)^2 + 1}$$

- *Bestimmung der Resonanzfrequenz*

$$\text{Im}\{\underline{Y}(\omega_0)\} = 0$$

$$\text{Im}\{\underline{Y}\} = \frac{-\omega L}{R_L^2 + (\omega L)^2} + \frac{\omega C}{(\omega C R_C)^2 + 1}$$



$$\frac{-\omega_0 L}{R_L^2 + (\omega_0 L)^2} + \frac{\omega_0 C}{(\omega_0 C R_C)^2 + 1} = 0$$

$$\frac{\omega_0 C}{(\omega_0 C R_C)^2 + 1} = \frac{\omega_0 L}{R_L^2 + (\omega_0 L)^2}$$

$$C(R_L^2 + (\omega_0 L)^2) = L((\omega_0 C R_C)^2 + 1)$$

$$C R_L^2 + \omega_0^2 L^2 C = \omega_0^2 C^2 L R_C^2 + L$$

$$\omega_0^2 (L^2 C - C^2 L R_C^2) = L - C R_L^2$$

$$\omega_0^2 = \frac{L - C R_L^2}{L^2 C - C^2 L R_C^2}$$

$$\omega_0 = \sqrt{\frac{L - C R_L^2}{L^2 C - C^2 L R_C^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_C^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_C^2}}$$

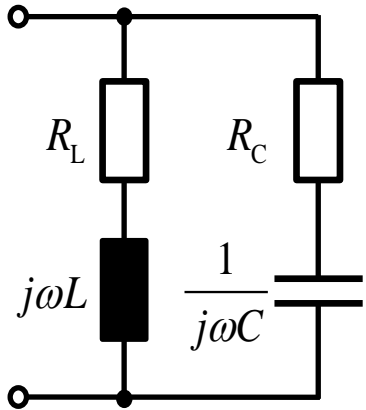
$$\omega_0 = \sqrt{\frac{L - CR_L^2}{L^2C - C^2LR_C^2}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{L - CR_L^2}{L - CR_C^2}}$$

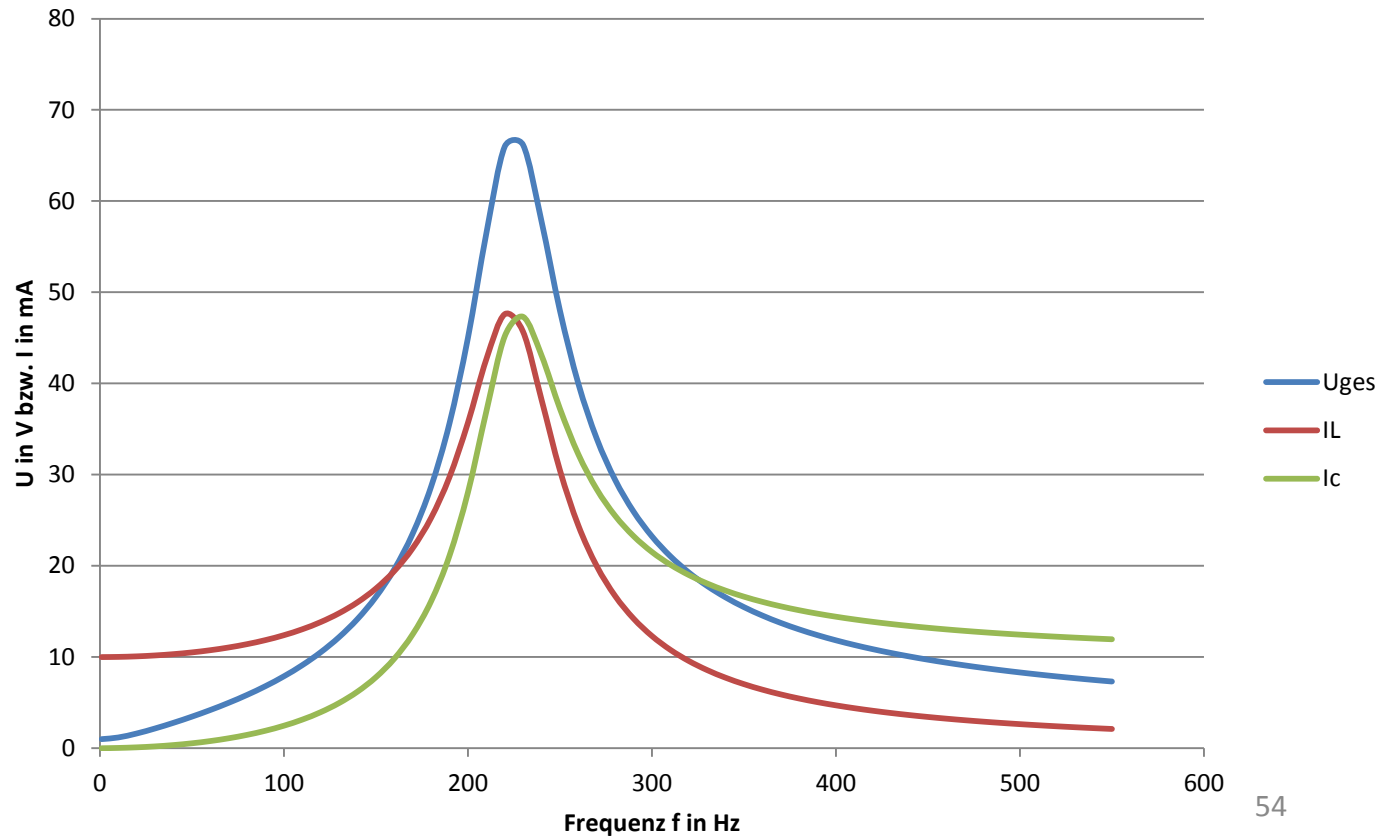
• *Untersuchungen zum Schwingverhalten*

1. Spezialfall:  $\frac{L}{C} > R_L^2$  und  $\frac{L}{C} > R_C^2$       oder       $\frac{L}{C} < R_L^2$  und  $\frac{L}{C} < R_C^2$

$\omega_0$  ist reell  $\rightarrow$  **Resonanz** = schwache Dämpfung (Schwingfall)



$L=1\text{H}$ ,  $C=0,5\mu\text{F}$   
 $R_L=100\Omega$ ,  $R_C=200\Omega$   
 $L/C=2000000$   
 $I_{\text{ges}}=10\text{mA}$   
 $f_0=226,8\text{Hz}$   
 $f_{0^*}$  (ohne  $R_L$  und  $R_C$ )  
 $=225,08\text{Hz}$

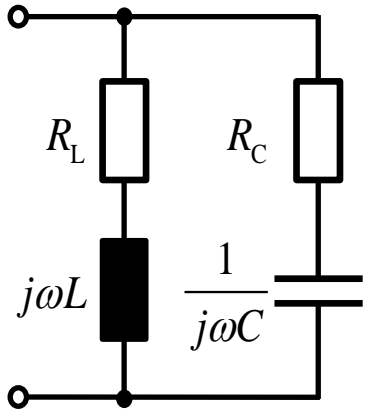




$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_C^2}}$$

2. Spezialfall:  $\frac{L}{C} < R_L^2$  und  $\frac{L}{C} > R_C^2$       oder       $\frac{L}{C} > R_L^2$  und  $\frac{L}{C} < R_C^2$

$\omega_0$  ist imaginär  $\rightarrow$  **keine Resonanz** = überkritische Dämpfung (Kriechfall)



$L=1\text{H}$ ,  $C=0,5\mu\text{F}$

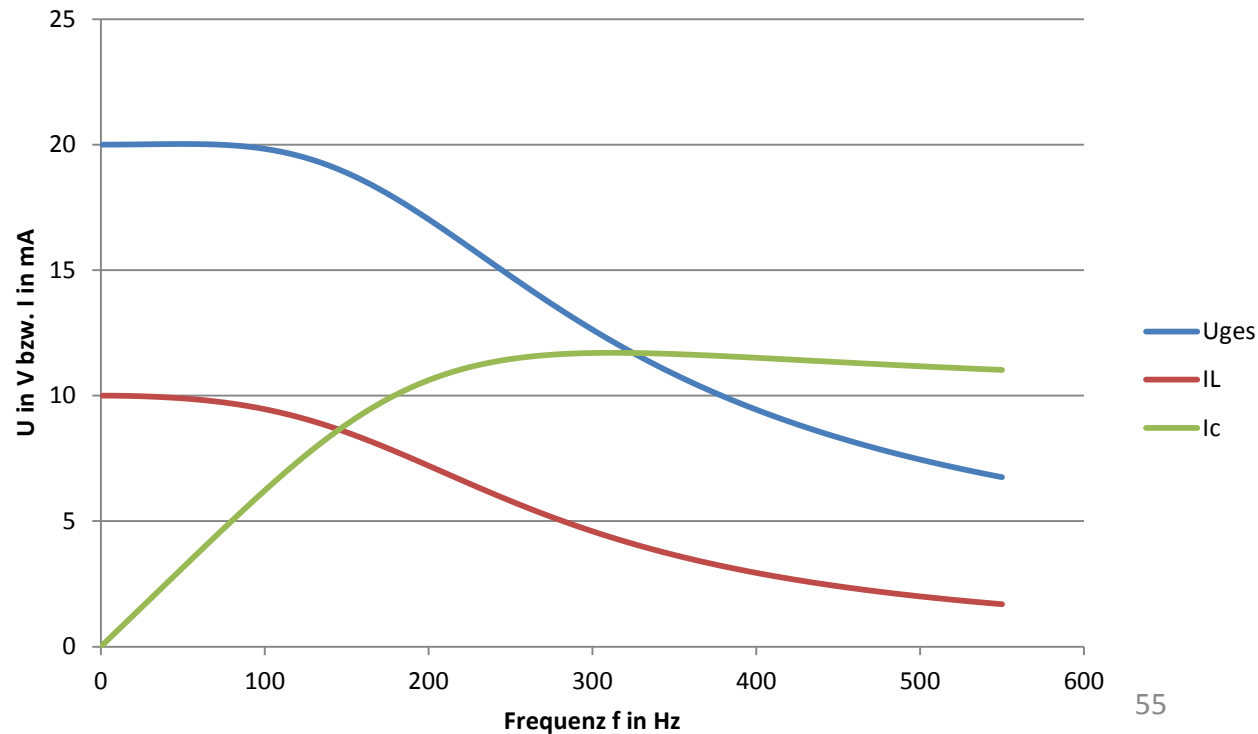
$R_L=2000\Omega$ ,

$R_C=200\Omega$

$L/C=2000000$

$I_{\text{ges}}=10\text{mA}$

Keine reelle Lösung  
für  $f_0$



$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{\frac{\frac{L}{C} - R_L^2}{\frac{L}{C} - R_C^2}}$$


---

3. Spezialfall:  $\frac{L}{C} = R_L^2$  und  $\frac{L}{C} = R_C^2$

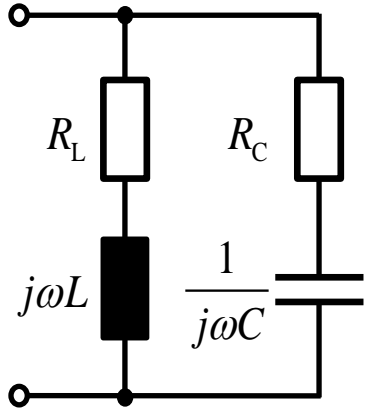
$$\operatorname{Im}\{\underline{Y}\} = \frac{-\omega L}{R_L^2 + (\omega L)^2} + \frac{\omega C}{(\omega C R_C)^2 + 1}$$

$$\operatorname{Im}\{\underline{Y}\} = \frac{-\omega L}{\frac{L}{C} + (\omega L)^2} + \frac{\omega C}{(\omega C)^2 \frac{L}{C} + 1}$$

$$\operatorname{Im}\{\underline{Y}\} = \frac{-\omega}{\frac{1}{C} + \omega^2 L} + \frac{\omega C}{\omega^2 C L + 1}$$

$$\operatorname{Im}\{\underline{Y}\} = \frac{-\omega C}{1 + \omega^2 L C} + \frac{\omega C}{\omega^2 C L + 1} = 0$$

Imaginärteil verschwindet für alle  $\omega \rightarrow$  ewige Resonanz



$L=1\text{H}, C=0,5\mu\text{F}$

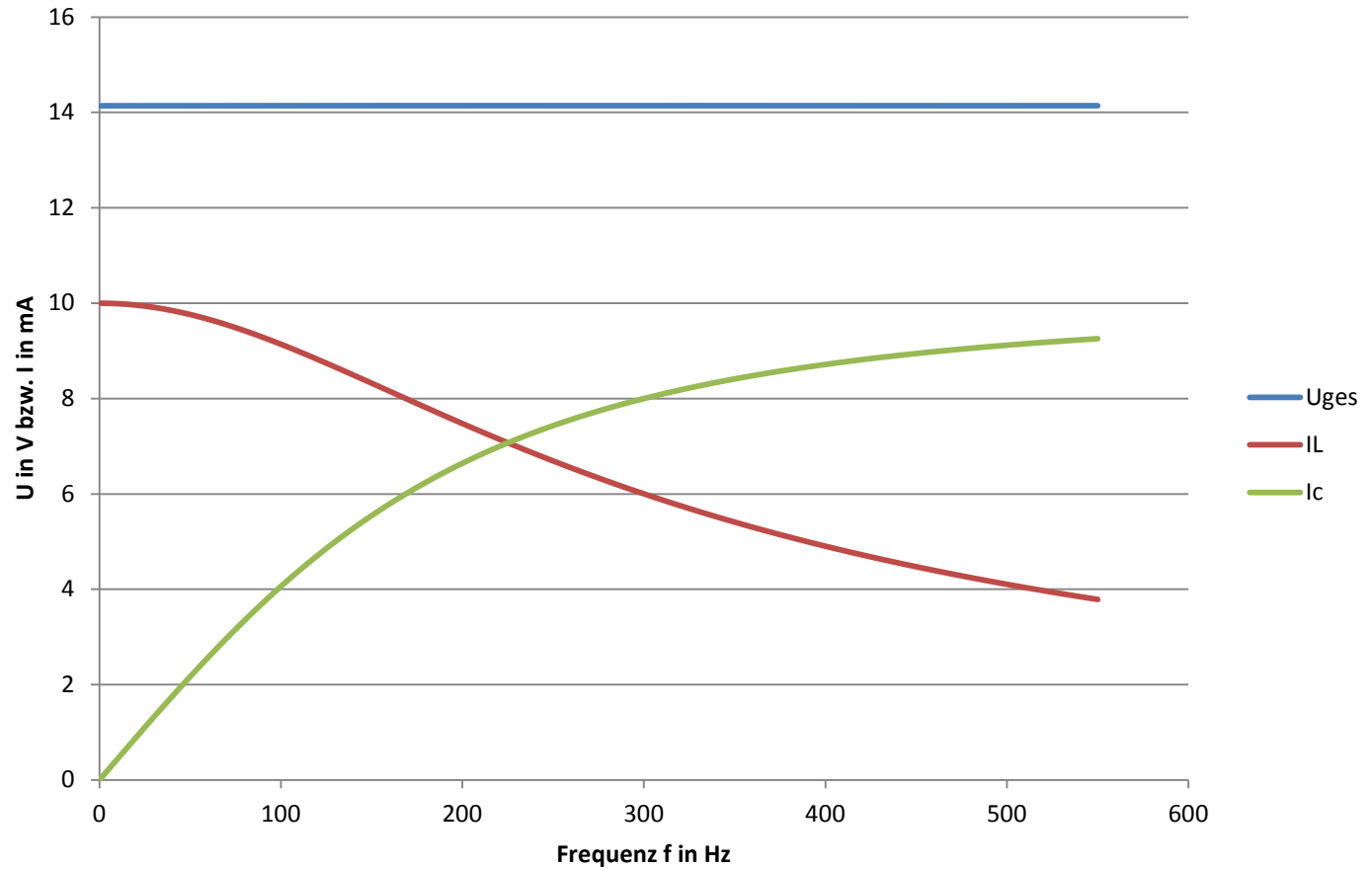
$R_L=1414\Omega,$

$R_C=1414\Omega$

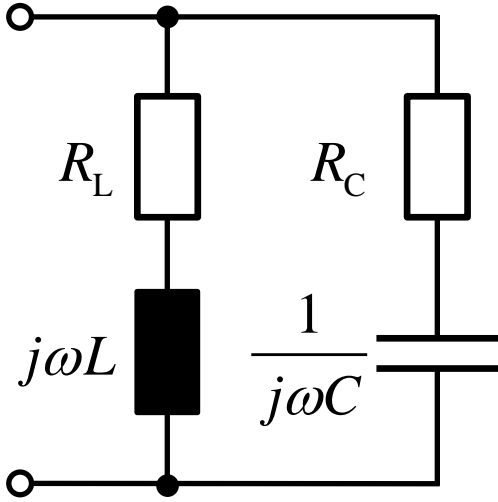
$L/C=2000000$

$I_{\text{ges}}=10\text{mA}$

Keine reelle Lösung  
für  $f_0$



- der Resonanzwiderstand bei  $R_C = 0$



$$\underline{Y} = \frac{1}{R_L + j\omega L} + \frac{1}{R_C + \frac{1}{j\omega C}}$$

$$\downarrow_{R_C=0} \underline{Y} = \frac{1}{R_L + j\omega L} + j\omega C = \frac{R_L - j\omega L}{R_L^2 + (\omega L)^2} + j\omega C$$

$$\underline{Y}(\omega_0) = \frac{R_L}{R_L^2 + (\omega_0 L)^2}$$

$$\omega_0^2 = \frac{L - CR_L^2}{L^2 C - C^2 LR_C^2}$$

$$\underline{Y}(\omega_0) = \frac{R_L}{R_L^2 + \frac{L - CR_L^2}{L^2 C} L^2} = \frac{R_L}{R_L^2 + \frac{L - CR_L^2}{C}}$$

$$\xrightarrow{R_C=0} \omega_0^2 = \frac{L - CR_L^2}{L^2 C}$$

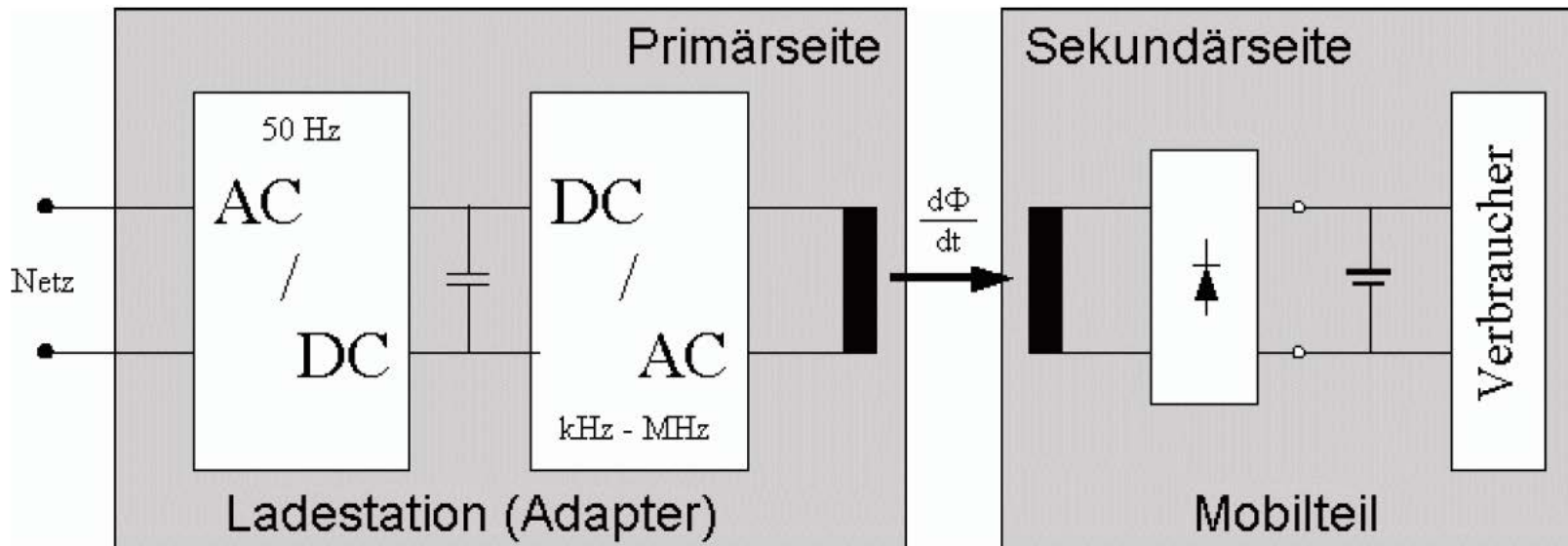
$$\underline{Y}(\omega_0) = \frac{R_L}{R_L^2 + \frac{L}{C} - R_L^2}$$

$$\underline{Y}(\omega_0) = \frac{R_L}{\frac{L}{C}} \quad \text{bzw.}$$

$$\underline{Z}(\omega_0) = \frac{1}{\underline{Y}(\omega_0)} = \frac{L}{CR_L}$$

# Anwendung: Drahtlose Ladetechnik

Blockschaltbild eines induktiven Ladegerätes



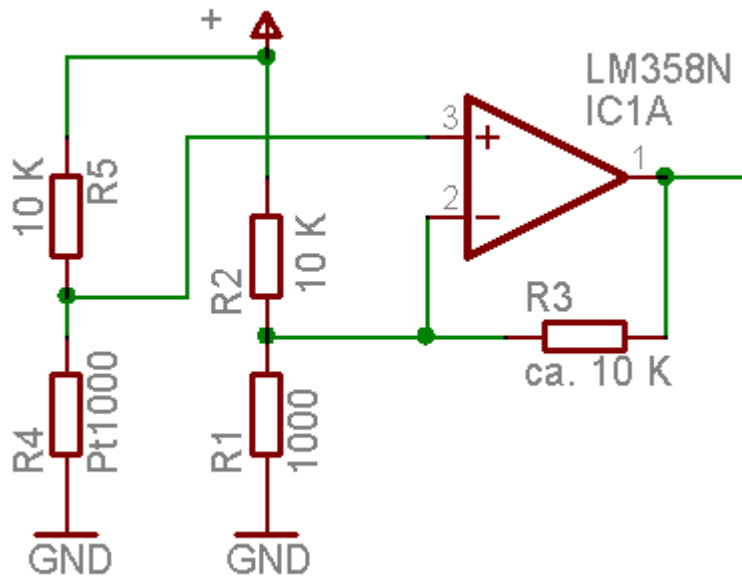
## Induktive Ladestation



## 7.2 Wechselstrommessbrückenschaltungen

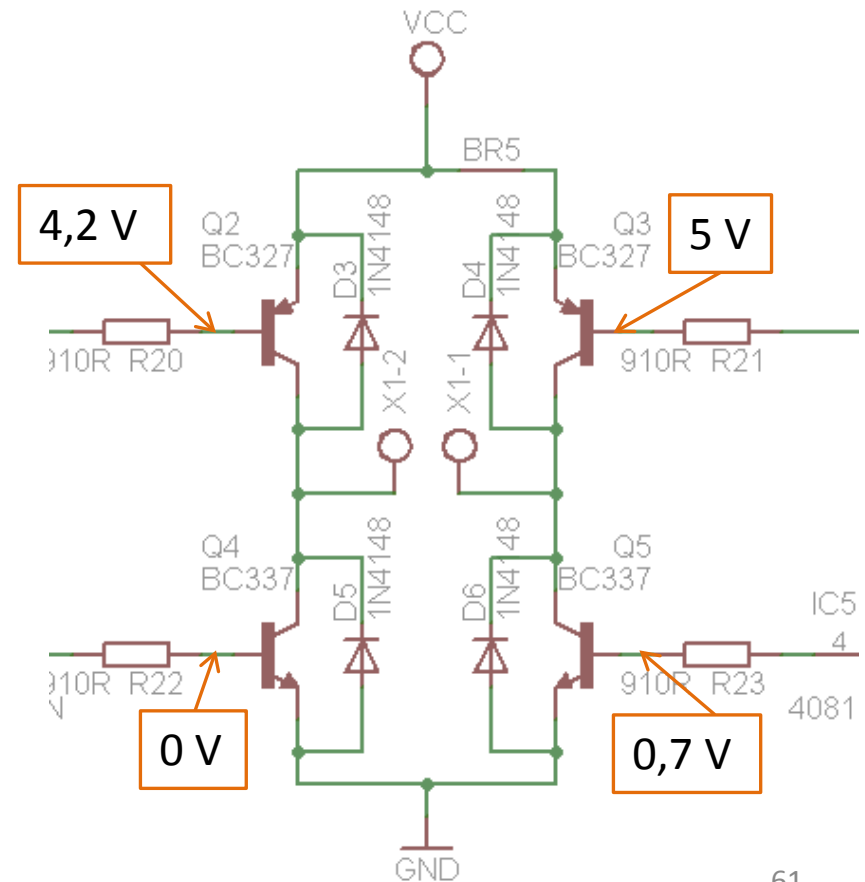
### Anwendung Brücken: Sensortechnik

(Temperaturmessung mit Pt1000)



### Anwendung Brücken: Motorsteuerung

(Potentiale für Rechtslauf)



# Vertiefung und Visualisierung:

**GETsoft**

LearnWeb

TaskWeb

LabWeb

TestWeb

BookWeb

über GETsoft

... stellt Elektrotechnik-Lernprogramme mit Kompendien, Beispiel- und Übungsaufgaben mit abgestufter Hilfe, Experimentierumgebungen, Glossarien u.v.m. zur Verfügung

GETsoft » LearnWeb » Messbrücken

Brückenkurs GET -  
Mathematik

Grundstromkreis

Gleichstromnetze

Ausgleichsvorgänge

Induktionsvorgänge

Frequenzselektive  
Schaltungen

Messbrücken

Transformator

Drehstromsystem

Fourier-Reihen

Fourier-Transformation

Laplace-Transformation

Leitungen

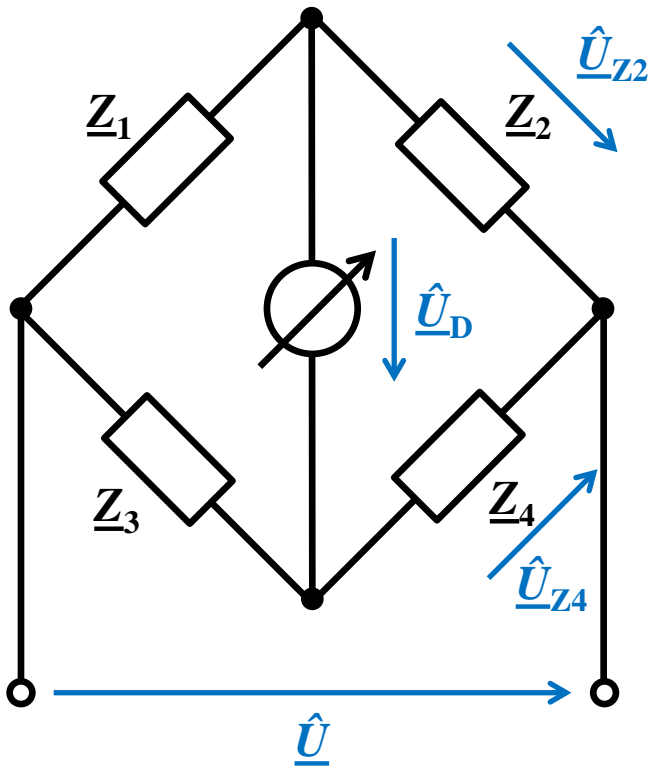
## Messbrücken



**Konzept:** Das Lernprogramm liefert ein Angebot von strukturiertem Kernwissen über Gleichstrom- und Wechselstrommessbrücken anhand:

- der Analyse der Brückenschaltung
- der Herleitung der Abgleichbedingungen bekannter Brückenanordnungen wie z.B. der Resonanzmessbrücke, der induktiven und kapazitiven Vergleichsbrücke, der Induktivitätsmessbrücke nach Maxwell-Wien u.a.
- von Aussagen zur Konvergenz des Abgleichs
- des Vergleichs der Anordnungen
- von Visualisierung durch Java-Applets





Abgleich:

$$\hat{U}_D = 0$$

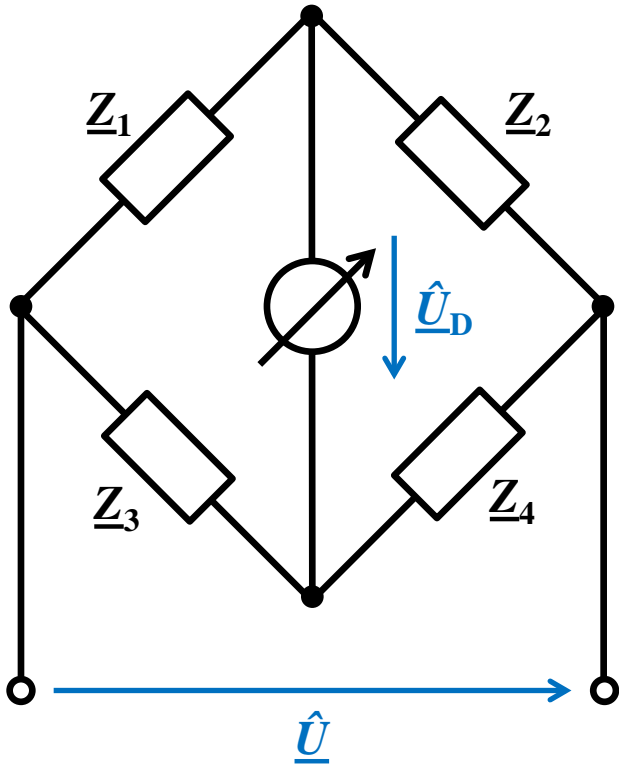
$$\hat{I}_D = 0$$

$$\hat{U}_{Z2} = \hat{U}_{Z4}$$

$$\hat{U}_{Z2} = \frac{Z_2}{Z_1 + Z_2} \hat{U}$$

$$\hat{U}_{Z4} = \frac{Z_4}{Z_3 + Z_4} \hat{U}$$

$$\hat{U}_{\underline{Z}_2} = \hat{U}_{\underline{Z}_4} \quad \hat{U}_{\underline{Z}_2} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \hat{U} \quad \hat{U}_{\underline{Z}_4} = \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} \hat{U}$$



$$\frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \hat{U} = \frac{\underline{Z}_4}{\underline{Z}_3 + \underline{Z}_4} \hat{U}$$

$$\underline{Z}_2(\underline{Z}_3 + \underline{Z}_4) = \underline{Z}_4(\underline{Z}_1 + \underline{Z}_2)$$

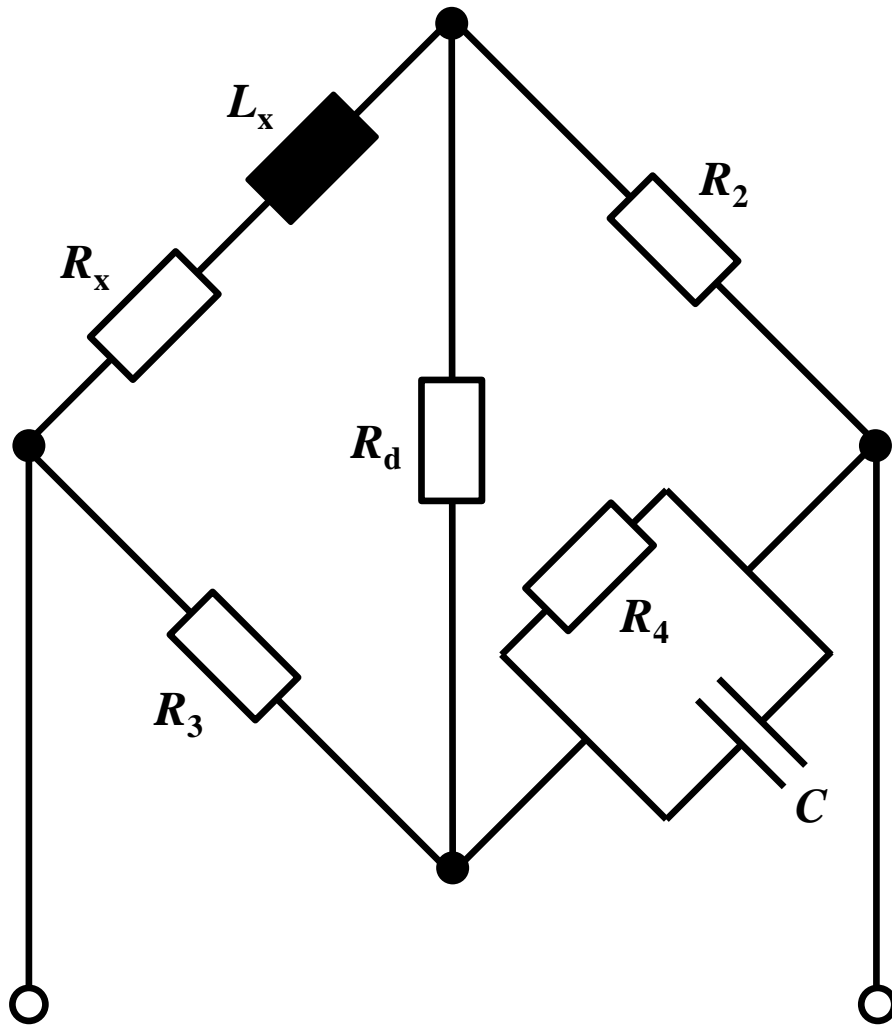
$$\underline{Z}_2\underline{Z}_3 + \underline{Z}_2\underline{Z}_4 = \underline{Z}_1\underline{Z}_4 + \underline{Z}_2\underline{Z}_4$$

$$\underline{Z}_2\underline{Z}_3 = \underline{Z}_1\underline{Z}_4$$

Abgleichbedingung:

$$\boxed{\frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_3}{\underline{Z}_4}}$$

## 7.2.1 Die Induktivitätsmessbrücke nach Maxwell-Wien



Abgleichbedingung:

$$\frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_3}{\underline{Z}_4}$$

$$\frac{R_x + j\omega L_x}{R_2} = R_3 \left( \frac{1}{R_4} + j\omega C \right)$$

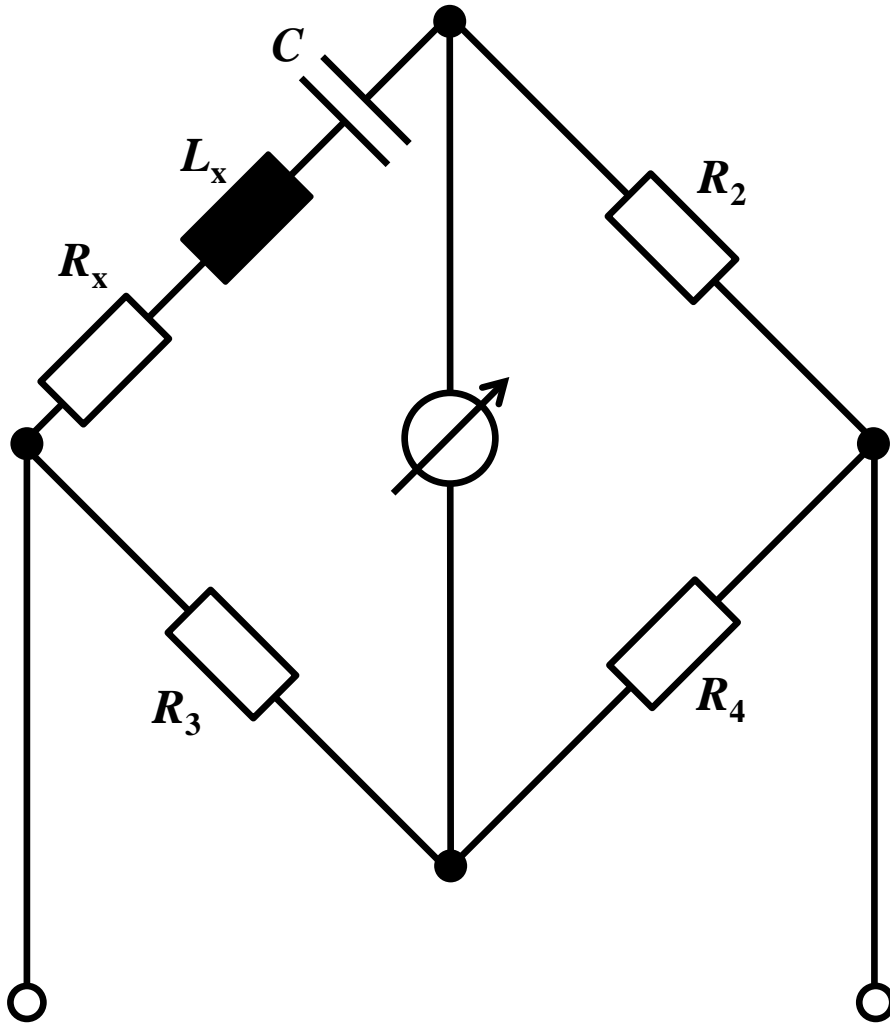
$$\frac{R_x}{R_2} = \frac{R_3}{R_4}$$

$$\frac{\omega L_x}{R_2} = \omega C R_3$$

$$R_x = \frac{R_3}{R_4} R_2$$

$$L_x = C R_2 R_3$$

## 7.2.2 Die Resonanzmessbrücke



Abgleichbedingung:

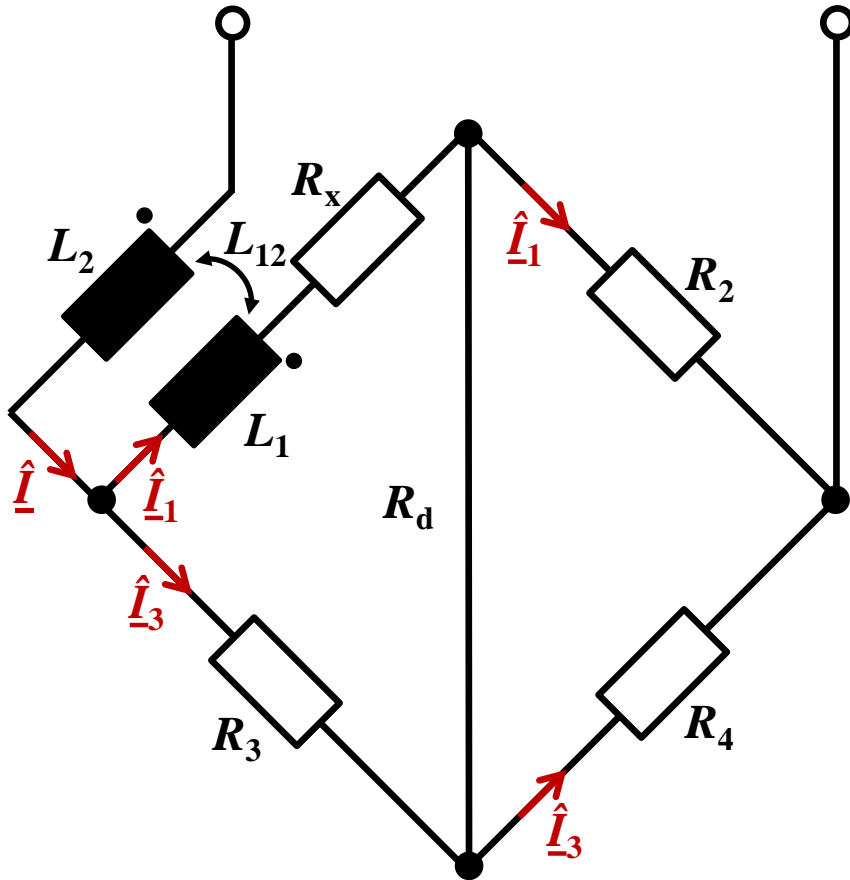
$$\frac{\underline{Z}_1}{\underline{Z}_2} = \frac{\underline{Z}_3}{\underline{Z}_4}$$

$$\frac{R_x + j\omega L_x + \frac{1}{j\omega C}}{R_2} = \frac{R_3}{R_4}$$

$$\frac{R_x}{R_2} = \frac{R_3}{R_4} \quad \omega L_x - \frac{1}{\omega C} = 0$$

$$R_x = \frac{R_2 R_3}{R_4} \quad L_x = \frac{1}{\omega^2 C}$$

## 7.2.3 Die Gegeninduktivitätsmessbrücke nach Wien



Abgleichbedingung:

$$\hat{I}_D = 0$$

$$\hat{U}_1 = \hat{U}_3 \quad \hat{U}_2 = \hat{U}_4$$

$$\hat{U}_1 = (R_1 + j\omega L_1) \hat{I}_1 - j\omega L_{12} \hat{I}_3$$

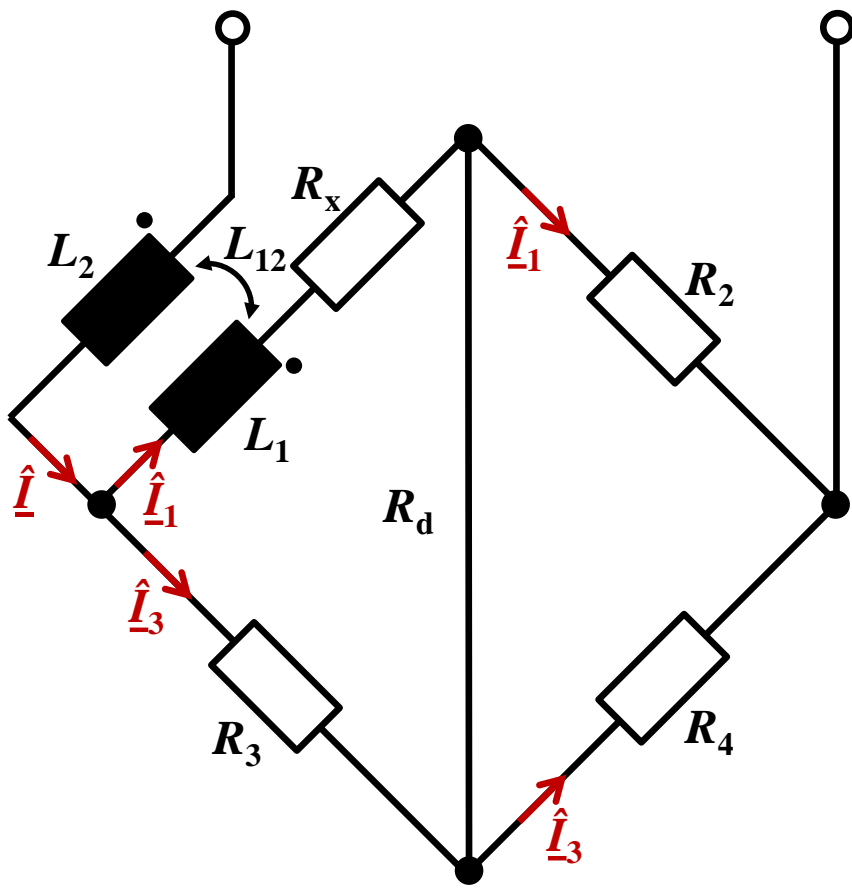
$$\hat{U}_3 = R_3 \hat{I}_3 \quad \hat{I} = \hat{I}_1 + \hat{I}_3$$

$$\hat{U}_2 = R_2 \hat{I}_1 \quad \hat{U}_4 = R_4 \hat{I}_3$$

$$(R_1 + j\omega L_1) \hat{I}_1 - j\omega L_{12} (\hat{I}_1 + \hat{I}_3) = R_3 \hat{I}_3$$

$$(R_1 + j\omega L_1 - j\omega L_{12}) \hat{I}_1 = (R_3 + j\omega L_{12}) \hat{I}_3$$

$$R_2 \hat{I}_1 = R_4 \hat{I}_3$$



$$(R_1 + j\omega L_1 - j\omega L_{12}) \hat{I}_1 = (R_3 + j\omega L_{12}) \hat{I}_3$$

$$R_2 \hat{I}_1 = R_4 \hat{I}_3$$

$$\frac{(R_1 + j\omega L_1 - j\omega L_{12}) \hat{I}_1}{R_2 \hat{I}_1} = \frac{(R_3 + j\omega L_{12}) \hat{I}_3}{R_4 \hat{I}_3}$$

$$\frac{(R_1 + j\omega L_1 - j\omega L_{12})}{R_2} = \frac{(R_3 + j\omega L_{12})}{R_4}$$

$$\frac{L_1 - L_{12}}{R_2} = \frac{L_{12}}{R_4}$$

$$\frac{L_1}{R_2} = \frac{R_2 + R_4}{R_2 R_4} L_{12}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{L_1}{R_2} = \left( \frac{1}{R_4} + \frac{1}{R_2} \right) L_{12}$$

$$L_{12} = \frac{R_4}{R_2 + R_4} L_1$$