

Klausur Mathe 2 Seite 2019

① ~~Kern~~ $\text{Ker}(L) = \{ \vec{x} \in \mathbb{R}^4 : L(\vec{x}) = \vec{0} \}$

\hookrightarrow homogenes LGS

$$\begin{array}{cccccc} 1 & -3 & 2 & -1 & 0 & \\ 2 & -6 & 3 & 1 & 0 & -2I \end{array}$$

$$\begin{array}{cccccc} 1 & -3 & 2 & -1 & 0 & +2II \\ 0 & 0 & -1 & 3 & 0 & -(-1) \end{array}$$

$$\begin{array}{cccccc} 1 & -3 & 0 & 5 & 0 & \\ 0 & 0 & +1 & -3 & 0 & \end{array}$$

$$x_4 =: t, \quad t \in \mathbb{R}$$

$$x_5 = x_2 =: s, \quad s \in \mathbb{R}$$

$$x_1 = 3s - 5t$$

$$x_2 = s$$

$$x_3 = +3t$$

$$x_4 = t$$

$$\hookrightarrow \text{Ker}(L) = \text{span} \left(\left\{ \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ 3 \\ 1 \end{pmatrix} \right\} \right)$$

$$\dim(\text{Ker}(L)) = 2$$

②

$$\begin{pmatrix} -1 & 1 \\ 4 & -3 \end{pmatrix}^{-1} X \begin{pmatrix} 7 & 11 \\ 5 & 8 \end{pmatrix} = \begin{pmatrix} -4 & -2 \\ -7 & -12 \end{pmatrix}$$

$$X = \begin{pmatrix} -1 & 1 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} -4 & -2 \\ -7 & -12 \end{pmatrix} \begin{pmatrix} 7 & 11 \\ 5 & 8 \end{pmatrix}^{-1}$$

$$\begin{pmatrix} 7 & 11 \\ 5 & 8 \end{pmatrix}^{-1} = \frac{1}{56-55} \begin{pmatrix} 8 & -11 \\ -5 & 7 \end{pmatrix}$$

$$X = \begin{pmatrix} -1 & 1 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} -4 & -2 \\ -7 & -12 \end{pmatrix} \begin{pmatrix} 8 & -11 \\ -5 & 7 \end{pmatrix}$$

$$X = \begin{pmatrix} -3 & -5 \\ 5 & 8 \end{pmatrix} \begin{pmatrix} 8 & -11 \\ -5 & 7 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}}}$$

$$\textcircled{3} \quad A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

$$\det(A - \lambda I) = (5 - \lambda) \begin{vmatrix} 4 - \lambda & 1 \\ 1 & 4 - \lambda \end{vmatrix} =$$

$$= (5 - \lambda) [(4 - \lambda)^2 - 1] = 0$$

$$\hookrightarrow \lambda_1 = 5$$

$$(4 - \lambda)^2 - 1 = 0$$

$$4^2 - 8\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 8\lambda + 15 = 0$$

$$\hookrightarrow$$

$$\lambda_{2,3} = 4 \pm \sqrt{16 - 15} = 4 \pm 1$$

$$\lambda_2 = 5 = \lambda_1$$

$$\lambda_3 = 3$$

$$\text{alg}(\lambda_1) = 2 \quad \text{alg}(\lambda_3) = 1$$

$$\text{mit } 1 \leq \text{geo}(\lambda) \leq \text{alg}(\lambda)$$

$$\text{gilt } \text{geo}(\lambda_3) = 1$$

$$\text{EW } ER(\lambda_1=5): (A-\lambda I)\vec{x}=\vec{0}$$

$$\begin{array}{cccc} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 \end{array}$$

$$\hookrightarrow x_1 = x_3, \quad x_2 \in \mathbb{R}$$

$$\hookrightarrow ER(\lambda_1=5) = \underline{\text{span}\left\{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right\}}$$

$$\text{geo}(\lambda_1=5) = 2$$

$$ER(\lambda_3=3):$$

$$\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{array} \begin{array}{l} \\ : (2) \\ -I, \text{ streichen} \end{array}$$

$$\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}$$

$$\hookrightarrow x_1 = -x_3$$

$$x_2 = 0$$

$$\hookrightarrow ER(\lambda_3=3) = \underline{\text{span}\left\{\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\right\}}$$

④

$$y' = -2xy + \cos(x)e^{-x^2}$$

homogene DGL:

$$y' = -2xy$$

Sonderlösung:

$$y = 0$$

für $y \neq 0$:

$$y' = -2xy$$

Trennung der Variablen

$$\int \frac{1}{y} dy = \int -2x dx$$

$$\ln|y| = -x^2 + C_0$$

$$C_0 \in \mathbb{R}$$

$$|y| = e^{-x^2 + C_0}$$

$$y_h = C e^{-x^2}$$

$C \in \mathbb{R}$ (mit Sonderlösung)

Spezielle Lösung:

y_s Ansatz: VdK

$$y_s = c(x) \cdot e^{-x^2} \quad y_s' = -2xy_s + \cos(x)e^{-x^2}$$

$$c'(x)e^{-x^2} + c(x)(-2xe^{-x^2}) = -2xc(x)e^{-x^2} + \cos(x)e^{-x^2}$$

$$c'(x)e^{-x^2} = \cos(x)e^{-x^2}$$

$$c'(x) = \cos(x)$$

$$c(x) = \int \cos(x) dx$$

$$c(x) = \sin(x)$$

$$\hookrightarrow y_s = \sin(x)e^{-x^2}$$

$$y = y_h + y_s = Ce^{-x^2} + \sin(x)e^{-x^2} \quad C \in \mathbb{R}$$

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$$a) y''' + 3y'' + 3y' + y = 0$$

$$\hookrightarrow \lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0$$

$$\Leftrightarrow (\lambda + 1)^3 = 0$$

~~1)~~

$$\hookrightarrow \lambda_1 = -1 \quad (3\text{-fach})$$

$$\hookrightarrow y_1 = e^{-x} \quad y_2 = x e^{-x} \quad y_3 = x^2 e^{-x}$$

$$y_h = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x}, \quad c_1, c_2, c_3 \in \mathbb{R}$$

$$b) f(x) = 2e^x$$

$$~~f''(x)~~ f'(x) = 2e^x$$

$$f''(x) = 2e^x$$

$$f'''(x) = 2e^x$$

$$2e^x + 3 \cdot 2e^x + 3 \cdot 2e^x + 2e^x = 16e^x$$

$$8e^x + 8e^x = 16e^x \quad \text{w. A.}$$

$$c) y = y_h + y_s = c_1 e^{-x} + c_2 x e^{-x} + c_3 x^2 e^{-x} + 2e^x$$

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$$b) \phi(\pi) = \frac{f(\pi-0) + f(\pi+0)}{2} \\ = \frac{2 + 0}{2} = \underline{\underline{1}}$$

$$a) \phi(x) = \sum_{k=-\infty}^{\infty} c_k e^{i \frac{2\pi}{T} kx}$$

$$\text{Hier: } T = 2\pi$$

$$c_k = \frac{1}{T} \int_{-\pi}^{\pi} f(x) e^{-i \frac{2\pi}{T} kx} dx$$

$$c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx} dx$$

$$= \frac{1}{2\pi} \left(\int_{-\pi}^0 0 dx + \int_0^{\pi} 2 e^{-ikx} dx \right)$$

$$= \frac{1}{2\pi} \left[\frac{2}{-ik} e^{-ikx} \Big|_0^{\pi} \right]$$

$$= \frac{1}{2\pi} \cdot \frac{2}{-ik} \left(e^{-ik\pi} - e^{-ik \cdot 0} \right)$$

$$= \frac{2}{2\pi \cdot (-ik)} \cdot \left((-1)^k - 1 \right)$$

$$= \frac{(-1)^k - 1}{-ik\pi} = \begin{cases} 0, & \text{für } k \text{ gerade} \\ \frac{2}{ik\pi} & \text{für } k \text{ ungerade} \end{cases}$$

~~c_0~~

$$c_0 = \frac{1}{2\pi} \int_0^{\pi} 2 dx = \frac{1}{2\pi} [2\pi - 0] = 1$$

$$\phi(x) = 1 + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \frac{2}{i\pi(2k+1)} e^{+ikx}$$

⑦ Unterraumkriterium:

$$\circ U \subseteq V$$

$$\circ U \neq \emptyset$$

$$\circ \forall u, v \in U, \forall \alpha, \beta \in \mathbb{R}: (\alpha u + \beta v) \in U$$

~~hier:~~

$$u := \begin{pmatrix} u_1 \\ 1 \end{pmatrix} \quad u_1 \in \mathbb{R}$$

$$v := \begin{pmatrix} v_1 \\ 1 \end{pmatrix} \quad v_1 \in \mathbb{R}$$

$$\alpha \begin{pmatrix} u_1 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} v_1 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha u_1 + \beta v_1 \\ \alpha + \beta \end{pmatrix} \notin U$$

es existiert $\alpha \neq \beta \neq 1$